CS 151 Complexity Theory **Spring 2025**

Problem Set 1

Out: April 3 *Due:* **April 10**

Reminder: you are encouraged to work in groups of two or three; however you must turn in your own write-up and note with whom you worked. You may consult the course notes and the optional text (Papadimitriou). The full honor code guidelines and collaboration policy can be found in the course syllabus.

Please attempt all problems. **Please turn in your solutions via Gradescope, by 1pm on the due date.**

1. Downward self-reducibility. For a language A, define

   $$A_{<n} = \{x \in A : |x| < n\}.$$

   Language A is said to be *downward self-reducible* if it is possible to determine in polynomial time if $x \in A$ using the results of queries of the form "is $y \in A_{<|x|}$?" The queries may be *adaptive*, meaning that the polynomial time procedure may choose later queries depending on the results of earlier ones. Show that every downward self-reducible language is in **PSPACE**.

2. Show that one of the following inequalities must hold: $\mathbf{L} \neq \mathbf{P}$ or $\mathbf{P} \neq \mathbf{PSPACE}$. Note that both are believed to be true, and no one knows how to prove either one is true.

3. Show that logspace reductions are closed under composition. Then use the same ideas to prove that if language A is **P**-complete, then $A \in \mathbf{L}$ implies $\mathbf{L} = \mathbf{P}$.

4. Use a padding argument to show that if $\mathbf{L} = \mathbf{P}$ then $\mathbf{PSPACE} = \mathbf{EXP}$.

5. Prove that $\mathbf{SPACE}(O(n)) \neq \mathbf{P}$. (Note that while this is an interesting result, it doesn't seem to shed any light on the major open questions $\mathbf{L} \stackrel{?}{=} \mathbf{P}$ and $\mathbf{P} \stackrel{?}{=} \mathbf{PSPACE}$). Hint: consider a language A and a padded version of A. How are the two languages related with respect to space? How are they related with respect to time?