PNAS PROFILE

Profile of Thomas Y. Hou

Jennifer Viegas

Working at the interface between numerics and applied analysis, Caltech mathematician Thomas Hou has solved challenging problems concerning fluid dynamics and other multiscale systems. His research has applications in materials science, biomedical engineering, multiphase fluid dynamics, among other fields. Hou is currently addressing a longstanding open question in mathematical fluid dynamics: whether smooth initial data for the 3D incompressible Euler equations can form a finite-time singularity. Dating back to polymath Leonhard Euler (1707–1783), these equations lie at the core of theoretical fluid mechanics. Hou's work in this area, reported in his Inaugural Article, provides key insights.

Thomas Hou, a professor of applied and computational mathematics at the California Institute of Technology, is one of the world's foremost applied mathematicians. His approach using integrated numerics and analysis has helped solve challenging geophysical problems once thought to be intractable. His work carries implications for the development of next-generation flow simulators and new clinical technologies. Elected to the National Academy of Sciences (NAS) in 2024, Hou is a leading figure in the study of partial differential equations (PDEs), which lie at the core of theoretical fluid mechanics. His Inaugural Article (1), coauthored with former graduate student Jiajie Chen, presents a computer-assisted proof of the Hou-Luo Blowup Scenario, which describes a potential finite-time singularity in solutions of the 3D Euler equations that describe the motion of incompressible, inviscid fluids. The findings may reshape researchers' understanding of turbulence in fluids and extreme fluid events.

Overcoming Hardship in China

Hou was born in Guangzhou, China. When he was six years old, during China's Cultural Revolution, he and his family were exiled to a rural village in southern Guangdong Province because of the overseas connections of his parents, both of whom were medical doctors. Hou recalls, "From 1969 to 1977, we endured extreme hardships and political discrimination, and children from families like mine were not eligible to apply for college." However, his parents and uncles provided encouragement, motivating him to excel in every subject throughout his formative years. Hou credits his high school math teacher, Fangzhen Zeng, with sparking his lifelong fascination with mathematics.

In 1977, communist leader Deng Xiaoping reinstated the national college entrance exam for the first time since the Cultural Revolution. Hou was one of three students in his high school who were recommended to take the exam. Based on his performance, Hou was admitted to South China University of Technology in 1978. "Although I initially wanted to study mechanical engineering, my strong math scores resulted in my admission to the department of applied mathematics—an unexpected turn that ultimately set the course for my entire future," Hou says.

Portrait of Thomas Hou. Image credit: Vicki Chiu (California Institute of Technology, Pasadena, CA).

Key Mentors

Hou recalls the dedication of professors at the university. Mathematician Wen Lu, in particular, influenced Hou, sharing stories about world-renowned scholars in his field. Hou says, "Professor Lu encouraged me; if I worked hard enough, I too could eventually join their ranks someday."

After earning a bachelor's degree in mathematics in 1982, Hou traveled to the University of California, Los Angeles, for Master's and PhD degrees in mathematics. Hou's doctoral advisor, mathematician Björn Engquist, provided guidance throughout his early career. Hou recently co-organized a conference at the University of Texas at Austin in honor of Engquist's 80th birthday. Another influence was Stanley Osher, who also offered guidance that shaped Hou's professional growth.

From 1987 to1989, Hou served as a postdoctoral researcher at the Courant Institute of Mathematical Sciences in New York,

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This is a Profile of a member of the National Academy of Sciences to accompany the member's Inaugural Article, e2500940122, in vol. 122, issue 27.

Published July 23, 2025.

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where he was mentored by Russell Caflisch, Peter Lax, and George Papanicolaou. Hou joined the Courant Institute's faculty in 1989 as an assistant professor before advancing to an associate professorship in 1992. In 1993, he accepted a similar role at Caltech, where he advanced to a full professorship in 1998 before earning his present title in 2004.

Euler and Navier–Stokes Equations

As a graduate student and postdoctoral fellow, Hou became interested in the Euler and Navier–Stokes equations; the latter deal with fluid viscosity. The mainstream view until 1990 held that the point-vortex method for 2D Euler equations was inherently unstable. Hou challenged this assumption and proved the method's stability and convergence (2). The finding laid a foundation for subsequent numerical approaches in water-wave and interface-fluid computations.

Hou developed a level set approach for tracking incompressible fluid interfaces with surface tension, all within a Eulerian framework (3). The method remains robust under large fluid deformations and has influenced modern multiphase flow simulations involving complex interface dynamics.

Resolving Fluid Interface Problems

Hou and colleagues formulated the small-scale decomposition framework that extracts the leading order operator and treats it explicitly (4). This leads to an extremely effective scheme for fluid interface problems with surface tension. The framework has been applied to modeling interfacial flows in multiphase fluid dynamics, materials science, and biomedical engineering. For a survey published in *Acta Numerica* in 1995, Hou examined this and other numerical techniques for free boundary problems, where an evolving interface must be determined as part of the solution (5).

In a 1997 article published in the journal *Physics of Fluids*, Hou and colleagues used numerical simulations to uncover three distinct flow regimes (6). The article received the 1998 François Frankiel Award from the American Physical Society in recognition of its significant contribution to fluid mechanics. Since then, Hou has also received The Society of Industrial and Applied Mathematics' James H. Wilkinson Prize in Numerical Analysis and Scientific Computing, The Ralph E. Kleinman Prize, and The City University of Hong Kong William Benter Prize in Applied Mathematics, among other honors.

Hou and colleagues later provided an overview of boundary integral methods for handling multiphase flows and interface-driven processes in materials science (7). They explained how such methods simplify simulations of surface tension, droplet mergers, and other complex interfacial phenomena.

Multiscale Finite Element Method

In 1997, Hou and postdoctoral associate Xiao-Hui Wu proposed the multiscale finite element method (MsFEM) for solving elliptic PDEs in highly heterogenous media, such as composite materials and porous media (8). The method captures essential fine-scale phenomena using local basis functions without requiring full-scale resolution throughout the entire domain. Hou says, "I often use the analogy of Google Maps to convey this concept—reconstructing the global landscape seamlessly from many local snapshots."

Hou and his team have twice extended the MsFEM. In 2013, they introduced the Generalized Multiscale Finite Element Method (GMsFEM). Its main innovation is a systematic enrichment process that uses local snapshots and spectral problems to capture global effects within each coarse element (9). GMsFEM has since become a powerful tool for complex multiscale simulations and is explored in the 2023 book *Multiscale Model Reduction: Multiscale Finite Element Methods and Their Generalizations* (10).

The second extension was reported in 2023, when Hou led the development of the Exponentially Convergent Multiscale Finite Element Method (ExpMsFEM) by incorporating adaptive edge basis functions to achieve exponential convergence in solving multiscale Helmholtz PDEs, which can be applied to various physical problems (11). The work solidified the growing consensus that carefully chosen local bases, adaptively enriched, can boost accuracy in multiscale and wave-dominated problems without incurring excessive computational costs. Some major oil companies, such as Chevron and SLB, have adopted a variant of the methods in their next-generation flow simulators.

Hou has also laid the groundwork for next-generation signal-processing applications. He and former doctoral student Zuoqiang Shi formulated a data-driven framework for time-frequency analysis that uses nonlinear optimization to learn an adaptive basis directly from the data (12). The framework automatically discovers intrinsic oscillatory modes tailored to the signal at hand.

3D Euler Singularity Research

Hou is now a leading figure in research on the Euler and Navier–Stokes equations. A longstanding goal of mathematicians is to determine whether or not a singularity—a point where a function becomes infinite, thus breaking down the equation—exists in the 3D Euler equations. The question is closely related to one of The Seven Millennium Problems selected by The Clay Mathematics Institute in 2000 (tied to the Navier–Stokes equation).

Approaching the blowup question from a Langrangian perspective, he and his team established localized nonblowup criteria for the 3D incompressible Euler equations (13). The criteria have been used to disprove blowup scenarios proposed by other researchers (14).

Hou and a colleague identified transport as playing a critical role in preventing singularities (15). Follow-up work provided further evidence that convection is a stabilizing force in 3D incompressible flows (16). In an *Acta Numerica* survey, Hou examined these findings and the overall problem of whether solutions to the 3D incompressible Euler and Navier–Stokes equations can develop finite-time singularities (17).

The Hou-Luo Blowup Scenario

In two seminal papers (18, 19), Hou and postdoctoral associate Guo Luo used a highly adaptive mesh refinement strategy to discover the Hou-Luo Blowup Scenario. The results suggest that solutions to the 3D incompressible

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Euler equations can develop a finite-time singularity. Hou bolstered that hypothesis by reporting a class of potentially singular solutions for the 3D axisymmetric Navier–Stokes equations (20).

Using high-precision numerics and refined PDE analysis, Hou and Chen provided rigorous justification of the Hou-Luo Blowup Scenario in 3D Euler flows (21, 22). Their work illustrates how carefully orchestrated local estimates, advanced numerical validation, and optimal transport techniques can address some of the most elusive questions in fluid dynamics.

Hou's Inaugural Article (1) reports a computer-assisted proof of finite-time singularity, an achievement that extends beyond the Euler problem to other challenging PDEs lacking explicit reference solutions or featuring significant nonlocal interactions. He is currently in the process of developing another computer-assisted proof strategy tailored to the Navier–Stokes equations.

Guiding Principles

In addition to his parents and mentors, Hou credits his wife, mathematician Yu-Chung Chang-Hou of Pasadena City College, as being a constant source of support and inspiration. He says, "From caring for our children and my mother during my research travels to introducing me to Buddhism, she has played a pivotal role in my life." He also expresses gratitude to the founding Dharma Master Hsuan Hua of the City of Ten Thousand Buddhas in Ukiah, the founding Dharma Master Hui Shen of the Ji-Guang Temple in Taiwan, and fellow South China University of Technology alumnus K.C. Choi, whose financial support helped Hou conduct much of his recent work.

Hou says, "In both my personal life and my research endeavors, I have found that focusing on the well-being of others brings a profound sense of peace and clarity. When we act from a place of altruism, we unlock our own potential to make meaningful contributions to science—and to life."

1. J. Chen, T. Y. Hou, Singularity formation in 3D Euler equations with smooth initial data and boundary. *Proc. Natl. Acad. Sci. U.S.A.* **122**, e2500940122 (2025).
2. J. Goodman, T. Y. Hou, J. Lowengrub, The convergence of the point vortex method for the 2-D Euler equations. *Comm. Pure Appl. Math.* **43**, 415–430 (1990).
3. Y. C. Chang, T. Y. Hou, B. Merriman, S. Osher, Eulerian capturing methods based on a level set formulation for incompressible fluid interfaces. *J. Comput. Phys.* **124**, 449–464 (1996).
4. T. Y. Hou, J. Lowengrub, M. Shelley, Removing the stiffness from interfacial flows with surface tension. *J. Comput. Phys.* **114**, 312–338 (1994).
5. T. Y. Hou, Numerical solutions to free boundary problems. *Acta Numer.* **4**, 335–415 (1995).
6. T. Y. Hou, J. Lowengrub, M. Shelley, The long-time motion of vortex sheets with surface tension. *Phys. Fluids* **9**, 1933–1954 (1997).
7. T. Y. Hou, J. Lowengrub, M. Shelley, Boundary integral methods for multi-phase problems in fluid dynamics and materials science. *J. Comput. Phys.* **169**, 302–362 (2001).
8. T. Y. Hou, X. H. Wu, A multiscale finite element method for elliptic problems in composite materials and porous media. *J. Comput. Phys.* **134**, 169–189 (1997).
9. Y. Efendiev, J. Galvis, T. Y. Hou, Generalized multiscale finite element methods. *J. Comput. Phys.* **251**, 116–135 (2013).
10. E. Chung, Y. Efendiev, T. Y. Hou, *Multiscale Model Reduction. Applied Mathematical Sciences* (Springer, New York, NY, 2023).
11. Y. Chen, T. Y. Hou, Y. Wang, Exponentially convergent multiscale methods for 2D high frequency heterogeneous Helmholtz equations. *Multiscale Model. Simul.* **21**, 849–883 (2023).
12. T. Y. Hou, Z. Shi, Data-driven time-frequency analysis. *Appl. Comput. Harmon. Anal.* **35**, 284–308 (2013).
13. J. Deng, T. Y. Hou, X. Yu, Geometric properties and the non-blowup of the three-dimensional Euler equation. *Commun. Partial Differ. Equations* **30**, 225–243 (2005).
14. T. Y. Hou, R. Li, Dynamic depletion of vortex stretching and non-blowup of the 3-D incompressible Euler equations. *J. Nonlinear Sci.* **16**, 639–664 (2006).
15. T. Y. Hou, C. Li, Dynamic stability of the 3D axisymmetric Navier–Stokes equations with swirl. *Commun. Pure Appl. Math.* **61**, 661–697 (2008).
16. T. Y. Hou, Z. Lei, On the stabilizing effect of convection for three-dimensional incompressible flows. *Commun. Pure Appl. Math.* **62**, 501–564 (2009).
17. T. Y. Hou, Blowup or no blowup? A unified computational and analytic approach to study 3-D incompressible Euler and Navier–Stokes equations. *Acta Numer.* **18**, 277–346 (2009).
18. G. Luo, T. Y. Hou, Potentially singular solutions of the 3D axisymmetric Euler equations. *Proc. Natl. Acad. Sci. U.S.A.* **111**, 12968–12973 (2014).
19. G. Luo, T. Y. Hou, Toward the finite-time blowup of the 3D incompressible Euler equations: A numerical investigation. *SIAM MMS* **12**, 1722–1776 (2014).
20. T. Y. Hou, The potentially singular behavior of the 3D Navier–Stokes equations. *Found. Comput. Math.* **23**, 2251–2299 (2023).
21. J. Chen, T. Y. Hou, Stable nearly self-similar blowup of the 2D Boussinesq and 3D Euler equations with smooth initial data. I: Analysis. arXiv [Preprint] (2023). https://doi.org/10.48550/arXiv.2305.05660 (Accessed 30 June 2025).
22. J. Chen, T. Y. Hou, Stable nearly self-similar blowup of the 2D Boussinesq and 3D Euler equations with smooth initial data. II: Rigorous numerics. *SIAM MMS* **23**, 25–130 (2025).

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