4 state aircraft problem with parametric uncertainty

Contents

Form the vector field and generate options

```
pvar x1 x2 x3 x4
x = [x1;x2;x3;x4];

f0 = [-0.24366*x2^3+0.082272*x1*x2+0.30492*x2^2+0.015426*x2*x3-0.082272*x2
      -0.054444*x2^2+0.10889*x2*x3-0.054444*x3^2+0.91136*x1-0.64516*x2-0.0
      x1;
      -0.864*x1-0.3211*x3];

fdX = [ 0.30765*x2^3+0.099232*x2^2+0.12404*x1+0.90912*x2+0.023258*x3-0.124
        0.00045754*x2;
        0;
        0];

fdm = [0;
       -0.054444*x2^2+0.10889*x2*x3-0.054444*x3^2-0.6445*x2-0.016621*x3+0.
       0;
       0];

fQ = [-0.0068307*x2^2-0.001428*x2;
      0;
      0;
      0];

pvar dX dm

f = f0+fdX*dX+fdm*dm+fQ*dm*dX;

dXrange = [0.99 2.05];
dmrange = [-0.1 0.1];
```

Iteration options and basis vector for V

```
zV = monomials(x,2:2);
Bis.flag = 0;
Bis.r1deg = 1;
```

Generate all options

```
[roaconstr,opt,sys] = GetRoaOpts(f, x, zV, [], Bis);
```

specify the range of parameters in the order they appear in sys.delvector

```
sys.delvector
```

```
ans =
  [ dX ]
  [ dm ]
```

```
ini_cell = [dXrange dmrange];
```

Modify the options

```
opt.coordoptim.IterStopTol = 1e-2;
opt.coordoptim.MaxIters = 20;
opt.display.roaest = 1;
opt.sim.NumConvTraj = 60;
opt.sim.dispEveryNth = 20;
opt.sim.flag = 1;
opt.sim.BetaMax = 8;
opt.sim.BetaInit = 8;
```

Robust ROA with deg(V) = 2

Let's first run a quick robust ROA analysis without B&B

Solve the problem and extract the solution

```
outputs = wrapper(sys,ini_cell,roaconstr,opt);
[V,beta,gamma,p,multip,betaUpper] = extractSol(outputs);
```

```
*** Start cellBetaCenter ***
No Prior V - Run Sim-Based Analysis
------------------Beginning simulations
System 1: Num Stable = 20  Num Unstable = 0  Beta for Sims = 8.000  Beta U
System 1: Num Stable = 40  Num Unstable = 0  Beta for Sims = 8.000  Beta U
System 1: Num Stable = 60  Num Unstable = 0  Beta for Sims = 8.000  Beta U
```

```
------------------End of simulations
------------------Begin search for feasible V
Try = 1   Beta for Vfeas = 8.000
------------------Found feasible V
Initial V (from the cvx outer bnd) gives Beta = 6.977
Pre-iteration V with Beta = 6.977 (Gamma = 0.919)
-------------------Iteration = 1
Beta = 8.250 (Gamma = 1.013)
-------------------Iteration = 2
Beta = 8.696 (Gamma = 1.045)
-------------------Iteration = 3
Beta = 8.895 (Gamma = 1.059)
-------------------Iteration = 4
Beta = 8.985 (Gamma = 1.065)
-------------------Iteration = 5
Beta = 9.036 (Gamma = 1.068)
-------------------Iteration = 6
Beta = 9.060 (Gamma = 1.070)
-------------------Iteration = 7
Beta = 9.076 (Gamma = 1.071)
-------------------Iteration = 8
Beta = 9.089 (Gamma = 1.072)
-------------------Iteration = 9
Beta = 9.100 (Gamma = 1.073)
-------------------Iteration = 10
Beta = 9.109 (Gamma = 1.073)
------- "Nominal" Beta =  9.109 ----
*** Verify at the vertices ***
****** Robust results:
Robust Beta = 3.949 (Nominal Beta = 9.109)
*** End of cellBetaCenter ***
```

Extract the solution

```
[V,beta,gamma,p,multip,betaUpper] = extractSol(outputs);
```

Certified beta without B&B

```
beta
```

```
beta =

    3.9487
```

B&B with deg(V) = 2

Change the options to run B&B

```
opt.display.BB = 1;
opt.BB.runBB = 1;
opt.BB.max_iter = 9;
```

Do not display the details within B&B steps

```
opt.display.roaest = 1;
```

Solve the problem and extract the solution

```
outputs = wrapper(sys,ini_cell,roaconstr,opt);
[V,beta,gamma,p,multip,betaUpper] = extractSol(outputs);
```

```
***Start B&B refinement***
-------Start cellBetaCenter for the initial partition --
*** Start cellBetaCenter ***
No Prior V - Run Sim-Based Analysis
------------------Beginning simulations
System 1: Num Stable = 20  Num Unstable = 0  Beta for Sims = 8.000  Beta U
System 1: Num Stable = 40  Num Unstable = 0  Beta for Sims = 8.000  Beta U
System 1: Num Stable = 60  Num Unstable = 0  Beta for Sims = 8.000  Beta U
------------------End of simulations
------------------Begin search for feasible V
Try = 1   Beta for Vfeas = 8.000
------------------Found feasible V
Initial V (from the cvx outer bnd) gives Beta = 6.361
Pre-iteration V with Beta = 6.361 (Gamma = 0.828)
-------------------Iteration = 1
Beta = 8.185 (Gamma = 0.961)
-------------------Iteration = 2
Beta = 8.741 (Gamma = 0.997)
-------------------Iteration = 3
Beta = 8.922 (Gamma = 1.008)
-------------------Iteration = 4
Beta = 9.000 (Gamma = 1.013)
-------------------Iteration = 5
Beta = 9.039 (Gamma = 1.016)
-------------------Iteration = 6
Beta = 9.062 (Gamma = 1.017)
-------------------Iteration = 7
Beta = 9.079 (Gamma = 1.018)
```

```
------------------Iteration = 8
Beta = 9.092 (Gamma = 1.019)
------------------Iteration = 9
Beta = 9.103 (Gamma = 1.020)
------------------Iteration = 10
Beta = 9.112 (Gamma = 1.021)
------- "Nominal" Beta =  9.112 ----
*** Verify at the vertices ***
****** Robust results:
Robust Beta = 3.943 (Nominal Beta = 9.112)
*** End of cellBetaCenter ***
-------End cellBetaCenter for the initial partition --
Current Beta  = 3.943,   Number of active cells = 1
-------Start B&B iteration number = 1 --
*** Start cellBetaCenter ***
No Prior V - Run Sim-Based Analysis
------------------Beginning simulations
System 1: Num Stable = 20  Num Unstable = 0  Beta for Sims = 8.000  Beta U
System 1: Num Stable = 40  Num Unstable = 0  Beta for Sims = 8.000  Beta U
System 1: Num Stable = 60  Num Unstable = 0  Beta for Sims = 8.000  Beta U
------------------End of simulations
------------------Begin search for feasible V
Try = 1   Beta for Vfeas = 8.000
------------------Found feasible V
Initial V (from the cvx outer bnd) gives Beta = 5.400
Pre-iteration V with Beta = 5.400 (Gamma = 0.699)
------------------Iteration = 1
Beta = 7.479 (Gamma = 0.879)
------------------Iteration = 2
Beta = 8.551 (Gamma = 0.949)
------------------Iteration = 3
Beta = 9.100 (Gamma = 0.979)
------------------Iteration = 4
Beta = 9.428 (Gamma = 0.996)
------------------Iteration = 5
Beta = 9.621 (Gamma = 1.007)
------------------Iteration = 6
Beta = 9.739 (Gamma = 1.014)
------------------Iteration = 7
Beta = 9.817 (Gamma = 1.018)
------------------Iteration = 8
Beta = 9.868 (Gamma = 1.021)
------------------Iteration = 9
Beta = 9.907 (Gamma = 1.024)
------------------Iteration = 10
...
```

Extract the solution

```
[V,beta,gamma,p,multip,betaUpper] = extractSol(outputs);
```

Certified beta with non B&B

```
beta
```

```
beta =

    5.2997
```

Plot the improvement in beta over B&B steps

```
dd = outputs.BBInfo.info(1);

for i1 = 1:opt.BB.max_iter
    act = dd(end).Active;
    ind = act == 1;
    B(i1) = min(dd.Beta_vec(ind));
    dd = outputs.BBInfo.info(i1*2+1);
end

plot(B,'*-');hold on;
xlabel('iteration number','fontsize',24)
ylabel('certified \beta','fontsize',24);
title('improvement in \beta for deg(V)=2','fontsize',24);



%
```