

## Robust ROA calculations

dynamics:

$$\dot{x}_1 = x_2;$$

$$\dot{x}_2 = -x_2 - 2x_1 + 2x_1^3 + \delta(-x_1 + x_1^3);$$

with  $\delta \in [-1, 1]$

This example was also used in Topcu and Packard, IEEE TAC, 2009 (in the special issue on positive polynomials in controls (example 1 in the paper))

```
% Form the vector field
pvar x1 x2;
x = [x1;x2];
x1dot = x2;
x2dot = -x2-2*x1+2*x1^3;
```

Nominal system

```
f = [x1dot; x2dot];
```

Introduce an uncertain parameter

```
pvar d1
```

Specify its range

```
ini_cell = [-1 1];
```

Form the uncertain vector field

```
f = f + d1*[0; -x1+x1^3];
```

```
% Get the vertex system
[roaconst, opt, sys] = GetRoaOpts(f, x);
[fNOM, fVER] = getf(sys, ini_cell);

% Generate the options, etc.
zV = monomials(x, 2:4);
Bis.flag = 0;
Bis.rldeg = 4;

[roaconst, opt, sys] = GetRoaOpts(fVER, x, zV, [], Bis);
sys.fWithDel = [];

opt.sim.NumConvTraj = 40;
opt.display.roaest = 1;
```

## Run the computations

```
outputs = wrapper(sys,[],roaconstr,opt);
```

```
-----Beginning simulations
System 1: Num Stable = 0      Num Unstable = 1      Beta for Sims = 3.289      Beta UB = 3.289
System 1: Num Stable = 0      Num Unstable = 2      Beta for Sims = 1.390      Beta UB = 1.390
System 1: Num Stable = 2      Num Unstable = 3      Beta for Sims = 1.306      Beta UB = 1.306
System 1: Num Stable = 4      Num Unstable = 4      Beta for Sims = 0.913      Beta UB = 0.913
System 1: Num Stable = 6      Num Unstable = 5      Beta for Sims = 0.861      Beta UB = 0.861
System 1: Num Stable = 12     Num Unstable = 6      Beta for Sims = 0.818      Beta UB = 0.842
System 1: Num Stable = 18     Num Unstable = 7      Beta for Sims = 0.777      Beta UB = 0.808
System 2: Num Stable = 1      Num Unstable = 1      Beta for Sims = 1.476      Beta UB = 0.808
System 2: Num Stable = 3      Num Unstable = 2      Beta for Sims = 1.402      Beta UB = 0.808
System 2: Num Stable = 6      Num Unstable = 3      Beta for Sims = 1.114      Beta UB = 0.808
System 2: Num Stable = 6      Num Unstable = 4      Beta for Sims = 1.058      Beta UB = 0.808
System 2: Num Stable = 8      Num Unstable = 5      Beta for Sims = 1.000      Beta UB = 0.808
System 2: Num Stable = 10     Num Unstable = 6      Beta for Sims = 0.929      Beta UB = 0.808
System 2: Num Stable = 11     Num Unstable = 7      Beta for Sims = 0.882      Beta UB = 0.808
-----End of simulations
-----Begin search for feasible V
Try = 1      Beta for Vfeas = 0.882
Try = 2      Beta for Vfeas = 0.838
-----Found feasible V
Initial V (from the cvx outer bnd) gives Beta = 0.173
-----Iteration = 1
Beta = 0.567 (Gamma = 0.535)
-----Iteration = 2
Beta = 0.665 (Gamma = 0.604)
-----Iteration = 3
Beta = 0.716 (Gamma = 0.640)
-----Iteration = 4
Beta = 0.739 (Gamma = 0.656)
```

## Extract the solution

```
[V,beta,gamma,p,multip,betaUpper] = extractSol(outputs);
```

```
beta
```

```
beta =
```

```
0.7388
```

## Upper bound on beta

```
betaUpper
```

```
betaUpper =
```

```
0.8822
```

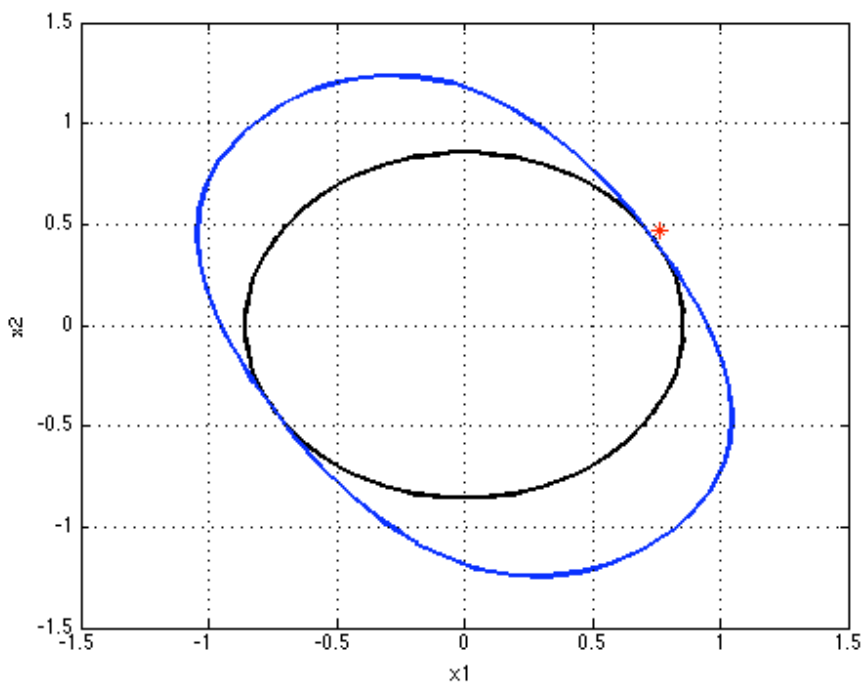
## Plot the results

```

[Cp4,hp4] = pcontour(p,beta,[-2 2 -2 2],'k'); hold on;
set(hp4,'linewidth',2);
[CV4,hV4] = pcontour(V,gamma,[-2 2 -2 2],'b');
set(hV4,'linewidth',2);
set(gca,'xlim',[-1.5 1.5],'ylim',[-1.5 1.5]);

traj = outputs.RoaEstInfo.info.SimLFG.sim.Trajectories(1).unstab(end).state;
pval = peval(traj,p.coef,p.deg);
[aux,ind] = min(pval);
plot(traj(1,ind),traj(2,ind),'r*','markersize',8);
grid on;

```



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