This is a final. You may consult only the course notes and the optional text (CLRS). You may not collaborate. The full honor code guidelines can be found in the course syllabus.

There are 4 problems on 2 pages. Please attempt all problems. To facilitate grading, please turn in each problem on a separate sheet of paper and put your name on each sheet. Do not staple the separate sheets. Good luck!

**Instructions for turning in the exam:** Please turn in your exams to Diane Goodfellow in Annenberg 246 before noon on Tuesday June 10.

1. Given subsets $S_1, S_2, \ldots, S_n$ of a universe $U$, and an integer $k$, a maximum $k$-cover is a collection of $k$ of the subsets that covers the maximum number of elements of $U$. Finding a maximum $k$-cover is NP-hard. Give a greedy approximation algorithm that achieves approximation ratio $\frac{e}{(e-1)}$. You may want to use the inequality $(1 - 1/x)^x \leq 1/e$.

2. Show how to find a maximum matching in a tree $G = (V, E)$ in time $O(|E|)$ operations (where here an “operation” includes arithmetic operations on integers of magnitude $O(|E|)$). Hint: use dynamic programming.

3. Given a list of integers $a_1, a_2, \ldots, a_n$ we are interested in finding a subsequence having maximum sum; i.e., if for $i \leq j$ we define $A_{i,j} = \sum_{i \leq k \leq j} a_k$, we want $i, j$ such that $A_{i,j}$ is maximum. Give a divide-and-conquer algorithm for this problem that uses $O(n \log n)$ operations, where here an “operation” includes arithmetic operations on integers of magnitude $O(n \max_i |a_i|)$.

4. Consider a directed graph $G = (V, E)$ with positive, integer capacities $c_e$ for each edge $e$. Here is an LP with variables $x_e$ for each edge $e \in E$ and $y_v$ for each $v \in V$ defined in terms of graph $G$ and two distinguished vertices $s$ and $t$, which represents the dual of the max-flow LP:

$$\text{minimize } \sum_{e \in E} c_e x_e \text{ subject to }$$

- $y_v - y_u + x_e \geq 0$ for all $e = (u, v) \in E$, and
- $y_s = 1$, and
- $y_t = 0$, and
- $x_e \geq 0$ for all $e \in E$.

You may assume that every vertex is reachable from $s$, and that every vertex can reach $t$.

(a) Prove that the constraint matrix for this LP is totally unimodular. Hint: for one case in the proof, consider adding the rows of a submatrix.

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(b) Prove that the optimum value of this LP equals the value of the minimum $s-t$ cut in graph $G$. 