CS 38 Introduction to Algorithms		**Spring 2014**
	Final	
Out: June 3		*Due:* **June 10**

This is a final. You may consult only the course notes and the optional text (CLRS). *You may not collaborate.* The full honor code guidelines can be found in the course syllabus.

There are 4 problems on 2 pages. Please attempt all problems. **To facilitate grading, please turn in each problem on a separate sheet of paper and put your name on each sheet. Do not staple the separate sheets.** Good luck!

Instructions for turning in the exam: Please turn in your exams to Diane Goodfellow in Annenberg 246 before noon on Tuesday June 10.

1. Given subsets $S_1, S_2, \ldots, S_n$ of a universe U, and an integer k, a *maximum k-cover* is a collection of k of the subsets that covers the maximum number of elements of U. Finding a maximum k-cover is NP-hard. Give a greedy approximation algorithm that achieves approximation ratio $e/(e-1)$. You may want to use the inequality $(1 - 1/x)^x \leq 1/e$.

2. Show how to find a maximum matching in a tree $G = (V, E)$ in time $O(|E|)$ operations (where here an "operation" includes arithmetic operations on integers of magnitude $O(|E|)$). Hint: use dynamic programming.

3. Given a list of integers $a_1, a_2, \ldots, a_n$ we are interested in finding a subsequence having maximum sum; i.e., if for $i \leq j$ we define $A_{i,j} = \sum_{i \leq k \leq j} a_k$, we want i, j such that $A_{i,j}$ is maximum. Give a divide-and-conquer algorithm for this problem that uses $O(n \log n)$ operations, where here an "operation" includes arithemetic operations on integers of magnitude $O(n|\max_i a_i|)$.

4. Consider a directed graph $G = (V, E)$ with positive, integer capacities c_e for each edge e. Here is an LP with variables x_e for each edge $e \in E$ and y_v for each $v \in V$ defined in terms of graph G and two distinguished vertices s and t, which represents the dual of the max-flow LP:

   minimize $\sum_{e \in E} c_e x_e$ subject to

   - $y_v - y_u + x_e \geq 0$ for all $e = (u, v) \in E$, and
   - $y_s = 1$, and
   - $y_t = 0$, and
   - $x_e \geq 0$ for all $e \in E$.

   You may assume that every vertex is reachable from s, and that every vertex can reach t.

   (a) Prove that the constraint matrix for this LP is totally unimodular. Hint: for one case in the proof, consider adding the rows of a submatrix.

(b) Prove that the optimum value of this LP equals the value of the minimum $s - t$ cut in graph G.