1. The key here is that $H$ is fixed. Suppose it has $k$ vertices. Given an input $G$, we can simply exhaustively try all potential subgraph isomorphisms. Specifically, we try every subset of $k$ vertices of $G$, and for each subset we try every one of the $k!$ possible ways of mapping it to the vertices of $H$. We can easily check, whether $H$ equals the subgraph induced by a given subset of $G$’s vertices (permuted according to a given mapping) – in fact this can be done in time polynomial in $k$.

Thus the overall running time of the algorithm is

$$\left(\frac{|G|}{|H|}\right) \cdot k! \cdot k^{O(1)}$$

Note that $\binom{n}{k} \leq n^k$, and $k! \leq k^k$, so we have an overall running time of $|G|^{O(k)}$. Since $k$ is fixed, this is polynomial in the size of the input, $G$.

2. Following the hint, we will build up a table $T$ which has a TRUE/FALSE entry for each $0 \leq B' \leq B$ telling us whether some multiset of the $x_i$ sum to exactly $B'$. Specifically, let $T$ be an array with $B+1$ entries. Initialize $T[i]$ to be FALSE for all $i$. For $B' = 0, 1, 2, \ldots B$ do:

- if some $x_i = B'$ then $T[B'] = TRUE$.
- else if for some $x_i$ we have $T[B' - x_i] = TRUE$, then set $T[B'] = TRUE$ (otherwise leave $T[B']$ set to FALSE).

At the end, we accept iff $T[B]$ is TRUE.

In each step we need to check all of the $x_i$, so the running time for one iteration of the loop is $O(n)$. The overall running time is $O(Bn)$ which is polynomial in the size of the input (here we use crucially that $B$ is presented in unary).

We should also argue that this algorithm is correct. We claim that after the $B'$-th iteration, all entries of $T$ up to and including $B'$ are correct (i.e. they are TRUE if there is a multiset of the $x_i$ summing to that value, and FALSE otherwise). The base case (when $B' = 0$) is clearly satisfied. Now assume $T$ is correct up to and including $B'-1$. There is a non-empty multiset summing to exactly $B'$ iff for some $i$, there is a non-empty multiset summing to $B' - x_i$. By induction $T[B' - x_i]$ is correct, and so we correctly fill in $T[B']$ in the $B'$-th iteration of the main loop.