1. A graph $G$ is called $k$-colorable if there is a way to assign a color to each vertex so that no edge has both endpoints assigned the same color, using at most $k$ distinct colors.

(a) Show that the language

$$2\text{-COLORABLE} = \{G : G \text{ is 2-colorable}\}$$

is in P by reducing it to a problem known to be in P.

(b) Show that the following language is NP-complete:

$$3\text{-COLORABLE} = \{G : G \text{ is 3-colorable}\}.$$ 

Hint: reduce from 3-SAT. Your graph will contain 1 vertex for each literal, and 3 special vertices connected in a triangle (which must then be colored with the three distinct colors). You may find this observation useful: in the following graph,

![Graph example](image)

if each of the grey nodes are colored with one of two colors, then it is possible to extend this coloring to a 3-coloring if and only if at least one of the three grey nodes on the left has the same color as the one on the right.

2. Let $(3, 3)$-SAT be the language consisting of satisfiable CNF formulas with at most 3 literals per clause, and at most 3 occurrences of any variable. Show that $(3, 3)$-SAT is NP-complete.
3. **max2sat** is the language consisting of all pairs $(\phi, k)$ where $\phi$ is a 2-CNF formula for which it is possible to simultaneously satisfy at least $k$ clauses. Show that **max2sat** is NP-complete. Hint: how many of the following clauses can be satisfied as a function of $x, y$ and $z$?

$$(x \lor x), (y \lor y), (z \lor z), (w \lor w),$$

$$(\neg x \lor \neg y), (\neg y \lor \neg z), (\neg z \lor \neg x),$$

$$(x \lor \neg w), (y \lor \neg w), (z \lor \neg w)$$