CS 21 Decidability and Tractability **Winter 2018**

Problem Set 4

Out: February 7 *Due:* **February 14**

Reminder: you are encouraged to work in groups of two or three; however you must turn in your own write-up and note with whom you worked. You may consult the course notes and the text (Sipser). The full honor code guidelines and collaboration policy can be found in the course syllabus.

Please attempt all problems. **To facilitate grading, please turn in each problem on a separate sheet of paper and put your name on each sheet. Do not staple the separate sheets.**

1. Two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are *isomorphic* if there is a bijection $f : V_1 \to V_2$ such that $(u, v) \in E_1 \Leftrightarrow (f(u), f(v)) \in E_2$. For a given graph H, define the following language:

$$\textsc{contains}_H = \{G : G \text{ contains a subgraph isomorphic to } H\}.$$

Here by "subgraph" we mean a subset of G's vertices together with all of G's edges on that subset of vertices – often called an "induced subgraph." Prove that for every H, CONTAINS_H is in P.

2. Show that the following problem is in P:

$$\textsc{unary subset sum} = \left\{(1^B, x_1, x_2, \ldots, x_n) : \exists \text{ a multiset } I \text{ of } [n] \text{ for which } \sum_{i \in I} x_i = B\right\}.$$

Here the x_i are all positive integers, as is B, and $[n]$ is shorthand for the set $\{1, 2, 3, \ldots, n\}$. The notation 1^B means a string of B ones, which is the representation of B in unary. Hint: solve the problem for all $B' \leq B$.