Reminder: you are encouraged to work in groups of two or three; however you must turn in your own write-up and note with whom you worked. You may consult the course notes and the text (Sipser). The full honor code guidelines and collaboration policy can be found in the course syllabus.

Please attempt all problems. Please turn in your solutions via Gradescope, by 1pm Pacific Time on the due date.

1. Two graphs \( G_1 = (V_1, E_1) \) and \( G_2 = (V_2, E_2) \) are isomorphic if there is a bijection \( f : V_1 \rightarrow V_2 \) such that \( (u, v) \in E_1 \iff (f(u), f(v)) \in E_2 \). For a given graph \( H \), define the following language:

\[
\text{CONTAINS}_H = \{ G : G \text{ contains a subgraph isomorphic to } H \}.
\]

Here by “subgraph” we mean a subset of \( G \)’s vertices together with all of \( G \)’s edges on that subset of vertices – often called an “induced subgraph.” Prove that for every \( H \), \( \text{CONTAINS}_H \) is in P.

2. Show that the following problem is in P:

\[
\text{UNARY SUBSET SUM} = \left\{ (1^B, x_1, x_2, \ldots, x_n) : \exists \text{ a multiset } I \text{ of } [n] \text{ for which } \sum_{i \in I} x_i = B \right\}.
\]

Here the \( x_i \) are all positive integers, as is \( B \), and \([n]\) is shorthand for the set \( \{1, 2, 3, \ldots, n\} \). The notation \( 1^B \) means a string of \( B \) ones, which is the representation of \( B \) in unary. Hint: solve the problem for all \( B' \leq B \).