

CS21
Decidability
and
Tractability

Lecture 1
January 4, 2023

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Outline

- administrative stuff
- motivation and overview of the course
- problems and languages
- Finite Automata

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Administrative Stuff

- Text: **Introduction to the Theory of Computation – 3rd Edition** by Mike Sipser
- Lectures self-contained
- Weekly homework
 - collaboration in small groups encouraged
 - separate write-ups (clarity counts)
- Midterm and final
 - indistinguishable from homework except cumulative, no collaboration allowed

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Administrative Stuff

- No programming in this course
- Things I assume you are familiar with:
 - programming and basic algorithms
 - asymptotic notation “big-oh”
 - sets, graphs
 - proofs, especially induction proofs

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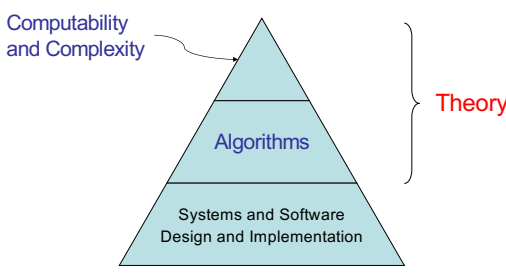
Motivation/Overview

- This course: introduction to **Theory of Computation**
 - what does it mean?
 - why do we care?
 - what will this course cover?

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Motivation/Overview



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Motivation/Overview

- At the heart of programs lie **algorithms**
- To study algorithms we must be able to speak **mathematically** about:
 - computational **problems**
 - **computers**
 - **algorithms**

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Motivation/Overview

- You might imagine that in principle
 - for each **problem** we would have an **algorithm**
 - the algorithm would be the fastest possible (requires **proof** that no others are faster)

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Motivation/Overview

- Our world (fortunately) is more interesting:
 - not all problems have algorithms (**we will prove this**)
 - for **many** problems we know embarrassingly little about what the fastest algorithm is
 - multiplying two integers
 - factoring an integer into primes
 - determining shortest tour of given n cities
 - for certain problems, fast algorithms would change the world (**we will see this**)

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Motivation/Overview

Part One:

computational problems, models of computation, characterizations of the problems they solve, and limits on their power

- Finite Automata and Regular Languages
- Pushdown Automata and Context Free Grammars

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Motivation/Overview

Part Two:

Turing Machines, and limits on their power (undecidability), reductions between problems

Part Three:

complexity classes P and NP, NP-completeness, limits of efficient computation

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Main Points of Course

(un)-decidability

Some problems have no algorithms!

(in)-tractability

Many problems that we'd like to solve have no **efficient** algorithms! (no one knows how to **prove** this yet...)

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What is a problem?

- Some examples:
 - given n integers, produce a sorted list
 - given a graph and nodes s and t , find the (first) shortest path from s to t
 - given an integer, find its prime factors
- problem associates each input to an output
- input and output are strings over a finite alphabet Σ

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What is a problem?

- A problem is a function:
 $f: \Sigma^* \rightarrow \Sigma^*$
- Simple. Can we make it simpler?
- Yes. Decision problems:
 $f: \Sigma^* \rightarrow \{\text{accept}, \text{reject}\}$
- Does this still capture our notion of problem, or is it too restrictive?

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What is a problem?

- Example: factoring:
 - given an integer m , find its prime factors
 $f_{\text{factor}}: \{0, 1\}^* \rightarrow \{0, 1\}^*$
- Decision version:
 - given 2 integers m, k , accept iff m has a prime factor $p < k$
- Can use one to solve the other and vice versa. True in general (homework).

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What is a problem?

- For most of this course, a problem is a decision problem:
 $f: \Sigma^* \rightarrow \{\text{accept}, \text{reject}\}$
- Equivalent notion: language
 $L \subseteq \Sigma^*$
the set of strings that map to “accept”
- Example: $L =$ set of pairs (m, k) for which m has a prime factor $p < k$

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