1. The East Model is a Markov chain with state space $\Omega = \{x = x_1 x_2 \ldots x_{n+1} \in \{0,1\}^{n+1} : x_{n+1} = 1\}$ with transitions defined as follows:

   Select a coordinate $i \in \{1, \ldots, n\}$ uar. If $x_{i+1} = 1$ then flip the value at i (i.e., replace x_i by $1 - x_i$), else do nothing.

   (a) Show that this Markov chain is irreducible and aperiodic, and that its stationary distribution is uniform.

   (b) Show that the mixing time is at least $n^2 - cn^{3/2}$ for some constant c.
   (Hint: consider the time to go from the state $\bar{0}1$ to any state of the form $1x_2 \ldots x_n 1$.
   You might need Chebyshev's inequality and the fact that a geometric random variable with parameter p has expectation $\frac{1}{p}$ and variance $\frac{1-p}{p^2}$.)

2. Let $1 \leq k \leq n/2$, and let Ω be the set of all subsets of $[n]$ of cardinality k. Consider the following Markov chain on Ω: from a subset S, pick an element $a \in S$ and an element $b \in [n] - S$, independently and uar, and move to the set $S + b - a$.

   (a) Show that this Markov chain (augmented if you like by a uniform self-loop probability) is ergodic with uniform stationary distribution.

   (b) Devise a coupling argument that shows that the mixing time is asymptotically $O(k \log k)$.