

1. Prove that for every $\varepsilon > 0$ there is a finite $\ell_0(\varepsilon)$ and an infinite sequence of bits a_1, a_2, \dots such that for every $\ell > \ell_0(\varepsilon)$ and every $i \geq 1$ the two binary vectors $u = (a_i, a_{i+1}, \dots, a_{i+\ell-1})$ and $v = (a_{i+\ell}, a_{i+\ell+1}, \dots, a_{i+2\ell-1})$ differ in at least $(1/2 - \varepsilon)\ell$ coordinates.