1. Give upper and lower bounds on the randomized query complexity of evaluating a boolean formula that is a complete binary tree of depth $n$ (= maximum number of edges on a root-leaf path) consisting of AND gates at even levels and OR gates at odd levels.

2. Let $L$ be any computational problem. In class we proved the equivalence $C_{\text{RAND}}(L) = C_{\text{DIST}}(L)$ for “Las Vegas” complexity. Prove the following extension of the weak side of this theorem to “Monte Carlo” complexity.

Definition: $C_{\text{RAND},\lambda}(L) = \inf_p \max_x E_p(C(r, x))$, where $p$ ranges over probability distributions on deterministic algorithms $r$, with the restriction on $p$ that for every $x$ (an input to the problem), the probability of error on $x$ is $\leq \lambda$. (Note that unlike in the Las Vegas case, $p$ is allowed to employ deterministic algorithms $r$ which err on some inputs $x$.)

Definition: $C_{\text{DIST},\lambda}(L) = \sup_q \min_r E_q(C(r, x))$, where $q$ ranges over all probability distributions on inputs $x$ and where $r$ ranges over all deterministic algorithms that have probability of error $\leq \lambda$ on distribution $q$.

As in class, $C(r, x)$ is the run-time (or query complexity, or any other real-valued performance measure) of $r$ on input $x$.

**Theorem 1 (Yao)** $C_{\text{RAND},\lambda}(L) \geq (1/2)C_{\text{DIST},2\lambda}(L)$. 