Theory

1. Consider a cubic B-spline with associated knot sequence $r_3 \leq r_2 \leq r_1 < s_1 \leq s_2 \leq s_3$, and let f be the polar form associated with the polynomial over $[r_1, s_1)$. Show how to construct the Bezier points $f(r_1, r_1, r_1)$, $f(r_1, r_1, s_1)$, $f(r_1, s_1, s_1)$, $f(s_1, s_1, s_1)$, from the de Boor points $f(r_3, r_2, r_1)$, $f(r_2, r_1, s_1)$, $f(r_1, s_1, s_2)$, $f(s_1, s_2, s_3)$.

2. **Bonus:** Provide an elementary proof that the recursive definition of uniform cardinal B-spline basis functions is equivalent to repeated convolutions with a box function. (Warning: we know of a proof, but it's not easy.)

Programming

1. Write a program to draw a cubic B-spline based on $k+1$ user-supplied de Boor points. Have the curve interpolate the first and last points by giving them multiplicity three; for example, by using the knot sequence $[0, 0, 0, 1, 2, \ldots, k-1, k, k, k]$, which effectively creates three coincident copies of each end point. Draw the curve by converting it to a sequence of Bezier curves (use problem 1 above and your code for Bezier curves).

2. Compute individual points along the curve corresponding to specified parameters in the range $[0, k]$ using the de Boor algorithm. Allow the parameter to be set interactively (perhaps with a slider), and indicate the point on the curve graphically.

3. **Bonus:** Extend your algorithm to work for B-splines of arbitrary degree.

Presentations and Projects

1. Select a paper (and several alternates) relating to geometric modeling that you would be interested in presenting to the class. You may choose papers from the class web page, from journals on computer graphics or computer aided design, or from any field in which geometric modeling or approximation theory plays an important role. Many papers will be on reserve in the computer science library in room 275 of Jorgensen.

2. Propose a tentative topic for a project. The project may include any mix of programming and theory. You may base the project on a specific paper or invent a project of your own. See the class web page for some suggestions.