SS/CS149 Introduction to Algorithmic Economics, Fall 2013

Homework # 2 **due: 11:59PM, November 24, 2013**

Your homework should be typed up and submitted by email to `ss149caltech AT gmail dotcom`, as a PDF attachment. **Please format your solutions so that each problem begins on a new page, and so that your name appears at the top of each page.**

You are strongly encouraged to collaborate with your classmates on homework problems, but each person must write up the final solutions individually. You should note on your homework specifically which problems were a collaborative effort and with whom. **You may not search online for solutions**, but if you do use research papers or other sources in your solutions, you must cite them.

Late policy: I will allocate each student 2 tokens at the beginning of the term. Each of these tokens can be used to buy a 24-hour extension on either homework during the term (you may spend them both on the same homework assignment). You should write clearly on your homework that you are using a token, and how many you are using. You cannot get extra tokens and zero credit will be given to late assignments.

Problems:

1. **Zero-Sum Games.**
   (a) Suppose a 2-player zero-sum game has two distinct Nash equilibria: (s_1, s_2) and (s_1', s_2'). Prove that (s_1', s_2) and (s_1, s_2') are also Nash equilibria of the game.
   (b) Is this "exchange" property also true in 2-player games that are not zero-sum? Prove it or give a counterexample.

2. **Stable Matching.** Consider a matching setting with 3 men (m_1, m_2, m_3) and 3 women (w_1, w_2, w_3), where each woman must be matched with exactly one man (and vice versa), with preferences as follows:

$$
\begin{aligned}
m_1 &: w_2 \succ w_1 \succ w_3 \\
m_2 &: w_1 \succ w_2 \succ w_3 \\
m_3 &: w_1 \succ w_2 \succ w_3 \\
w_1 &: m_1 \succ m_3 \succ m_2 \\
w_2 &: m_3 \succ m_1 \succ m_2 \\
w_3 &: m_1 \succ m_2 \succ m_3
\end{aligned}
$$

   (a) What are the two stable matchings, given the above preferences?
   (b) For each of the two stable matchings, construct a false set of preferences for an agent who prefers the other matching such that substituting those false preferences would result in her preferred matching being the only stable matching.

(c) Use the above to complete a proof that there does not exist any algorithm that always outputs a stable matching such that it is always a dominant strategy for every agent to report her true preferences.

3. **ϵ-Domination.** Say an action s_i of a player i is ϵ-*dominated* by action s'_i if for all strategy profiles s_{-i} of the other players, $u_i(s_i, s_{-i}) \leq u_i(s'_i, s_{-i}) - \epsilon$.

   (a) Let s_i be an ϵ-dominated action of a player i. Show that if player i uses the weighted majority algorithm discussed in class to choose her strategies in repeated play of the game, then the probability $\pi(s_i)$ that she is playing action s_i goes to zero as time goes to infinity (no matter how the other players choose their actions at each time step).

   (b) Can there be a correlated equilibrium in which some player plays an ϵ-dominated action with positive probability? Prove your answer.

4. **Payments and Truth.** In this problem, we will complete the proof that any truthful mechanism on a single-parameter domain where $\forall i$, $p_i(0, p_{-i}) = 0$, must have payment rule

$$p_i(v) = v_i \cdot w_i(x(v)) - \int_0^{v_i} w_i(x(z, v_{-i}))\, \mathrm{d}z.$$

   (a) Fix a player i, and opponents' stated bids b_{-i}. Write player i's utility as a function of her stated bid b_i, in terms of her valuation parameter v_i, her public summarization function w_i, the allocation rule x, and the payment rule p_i for player i.

   (b) Give the partial derivative of this utility function with respect to b_i. Note the constraints that truthfulness places on this partial derivative.

   (c) Finish the proof. Along the way, you'll do some integration and some rearranging of terms, and you'll need to use the assumption that $p_i(0, p_{-i}) = 0$.