SS/CS149 Introduction to Algorithmic Economics, Fall 2013

Homework # 1 **due: 11:59PM, October 25, 2013**

Your homework should be typed up and submitted by email to `ss149caltech AT gmail dotcom`, as a PDF attachment. **Please format your solutions so that each problem begins on a new page, and so that your name appears at the top of each page.**

You are strongly encouraged to collaborate with your classmates on homework problems, but each person must write up the final solutions individually. You should note on your homework specifically which problems were a collaborative effort and with whom. **You may not search online for solutions**, but if you do use research papers or other sources in your solutions, you must cite them.

Late policy: I will allocate each student 2 tokens at the beginning of the term. Each of these tokens can be used to buy a 24-hour extension on either homework during the term (you may spend them both on the same homework assignment). You should write clearly on your homework that you are using a token, and how many you are using. You cannot get extra tokens and zero credit will be given to late assignments.

Problems:

1. **Elimination of Dominated Strategies: Not Strict.**

   (a) Give a payoff matrix for a two player game (wherein player utilities are higher for greater values in the matrix) on which iterated deletion of dominated strategies (though not necessarily of *strictly* dominated strategies) can lead to two different single-celled outcomes (i.e., in each case, only a single strategy profile survives), depending on the sequence of deletions chosen.

   (b) For each of the two outcomes, give a sequence of deletions that leads to it.

   (c) What would iterated deletion of *strictly* dominated strategies yield on this game? Briefly explain why.

2. **Elimination of Dominated Strategies: Strict.** Prove that if only a single strategy profile s (i.e., a single cell of the game matrix) survives iterated deletion of strictly dominated strategies on a two-player game matrix, then s is the unique pure strategy Nash equilibrium of the game.

3. **Channel Sharing.** Consider a channel sharing game wherein n players are sharing a communication channel of capacity 1. Each player $i \in [n]$ selects an amount of flow $x_i \in [0, 1]$ to send on the channel (i.e., $\forall i, A_i = [0, 1]$ is the action set for player i). Given a profile of actions $x \in A$, for each i, player i has utility $u_i(x_i, x_{-i}) = x_i(1 - \sum_{j=1}^{n} x_j)$.

   (a) Is this game an exact potential game? Give a proof.

(b) Give a Nash equilibrium of this game, and prove that it is indeed an equilibrium.

(c) What is the social cost at the equilibrium you described?

(d) What is the social cost of the optimal outcome? Justify your answer (how do you know that the outcome you analyzed is optimal?).

4. **Congestions and Prisoners.** Consider the prisoners' dilemma game with the cost matrix below (e.g., when player 1 stays silent and player 2 confesses, player 1 incurs a cost of 5 and player 2 incurs a cost of 1):

P1 \ P2	Confess	Silent
Confess	4, 4	1, 5
Silent	5, 1	2, 2

Does this game have an equivalent congestion game? Give a proof of your answer.