## Relational Database

## System Implementation

CS122 - Lecture 9
Winter Term, 2018-2019

## Last Time: Plan Costing

- Introduced the notion of plan costing
- Goal: Faster plans end up with lower cost than slower ones
- Need to collect statistics on tables in order to make cost estimates
- A basic, minimal set of statistics:
- $n_{r}$ - the number of tuples in table $r$
- $b_{r}$ - the number of blocks containing tuples in $r$
- $l_{r}$ - the average size of a tuple in $r$, in bytes
- $\mathrm{V}(A, r)$ - number of distinct values of $A$ in table $r$
- $\min (A, r)$ - minimum value of $A$ in table $r$
- $\max (A, r)$ - maximum value of $A$ in table $r$


## Select Costs

- $\sigma_{\theta}(r)$
- Estimate number of rows produced $n_{\sigma}=n_{r} \times \mathrm{P}(\theta)$
- $\mathrm{P}(\theta)$ is the selectivity of the predicate
- i.e. the likelihood that a tuple will satisfy the predicate
- Simply need to estimate the selectivity of the predicate, then we can estimate the number of rows produced
- For now, assume that $r$ is a heap file
- Select operation will [almost] always read all blocks in $r$
- (Other file organizations and indexes change this...)


## Selectivity of Simple Predicates

- $\sigma_{A \leq v}(r)$
- Without a histogram, use minimum/maximum values for $A$ to estimate selectivity
- If $v<\min (A, r)$ :
- $\mathrm{P}(A \leq v)=0$
- If $v>\max (A, r)$ :
- $\mathrm{P}(A \leq v)=1$
- If $\min (A, r) \leq v \leq \max (A, r)$ :
- $\mathrm{P}(A \leq v)=(v-\min (A, r)) /(\max (A, r)-\min (A, r))$
- $\sigma_{A \geq v}(r)$ is similar


## Selectivity of Simple Predicates (2)

- $\sigma_{A=v}(r)$
- Assume uniform distribution of different values of $A$
- Estimate $\mathrm{P}(A=v)$ to be $1 / \mathrm{V}(A, r)$
- Estimate $n_{\sigma}=n_{r} / \mathrm{V}(A, r)$
- What if $A$ is a primary key for $r$ ?
- In that case, $\mathrm{V}(A, r)$ will be $n_{r}$
- $\mathrm{P}(A=v)$ will be $1 / n_{r}$, and $n_{\sigma}$ will be 1


## Selectivity of Simple Predicates (3)

- $\sigma_{A=v}(r)$
- If $A$ is a primary key for $r$, can also improve file-scan performance:
- Each value of $A$ can only appear once...
- Stop scanning $r$ when we find the specified row
- Average-case block-reads $=b_{r} / 2$; worst-case $=b_{r}$


## Selectivity of Simple Predicates (4)

- For inverse of these predicates: $\sigma_{A>v}(r), \sigma_{A \neq v}(r)$
- Simply compute selectivity as $1-\mathrm{P}(A \leq v)$ or $1-\mathrm{P}(A=v)$
- Boolean negation can be handled in similar way:
- $\sigma_{\neg \theta}(r)$
- Simple: $\mathrm{P}(\neg \theta)=1-\mathrm{P}(\theta)$


## Complex Selects

- If a predicate includes multiple conditions, estimate selectivities of the components, then combine
- Conjunctive selections: $\sigma_{\theta 1 \wedge \theta 2 \wedge \ldots} \ldots(r)$
- Assumption: conditions are independent of each other
- $\mathrm{P}(\theta 1 \wedge \theta 2 \wedge \ldots)=\mathrm{P}(\theta 1) \times \mathrm{P}(\theta 2) \times \mathrm{P}(\ldots)$
- Disjunctive selections: $\sigma_{\theta 1}$ v $\theta 2 \mathrm{v}$... $(r)$
- Again, compute selectivities of components
- $\mathrm{P}(\theta 1 \vee \theta 2 \vee \ldots)=$ probability that a tuple satisfies at least one condition $=1-$ probability it satisfies none of them
- $\mathrm{P}(\theta 1 \vee \theta 2 \vee \ldots)=1-(1-\mathrm{P}(\theta 1)) \times(1-\mathrm{P}(\theta 2)) \times \ldots$


## Estimating Selectivity

- One major assumption here:
- Conditions involve simple comparisons between an attribute and a constant
- Frequently not true!
- SELECT * FROM employees WHERE salary * 1.05 > 100000;
- DELETE FROM employees WHERE compute_popularity(emp_id) < 20;
- In simpler cases, can analyze expression to make estimate
- For more difficult situations, use default selectivities, e.g.
- $1 / 2$ when it's expected to be "common" for tuples to satisfy the condition
- $1 / 3$ or $1 / 4$ when it's expected to be "uncommon" or "rare"


## Selection Against Subplans

- Previous examples were all against a relation $r$
- We had statistics for $r$ !
- Plans often contain selections against subplans
- Need to estimate the statistics of a plan-node's result as well, if higher-level cost estimates will be useful
- Most difficult are $\mathrm{V}(A, r), \min (A, r)$, and $\max (A, r)$
- If selection involves an equality: $\sigma_{A=v}(r)$
- $\mathrm{V}\left(A, \sigma_{A=v}\right)=1$
- $\min \left(A, \sigma_{A=v}\right)=\max \left(A, \sigma_{A=v}\right)=v$


## Selection Against Subplans (2)

- If selection involves a comparison: $\sigma_{A \leq v}(r)$
- Assume $\min (A, r) \leq v \leq \max (A, r)$
- $\min \left(A, \sigma_{A \leq v}\right)=\min (A, r)$
- $\max \left(A, \sigma_{A \leq v}\right)=v$
- Estimate $V\left(A, \sigma_{A \leq v}\right)$

$$
\begin{aligned}
& =\mathrm{V}(A, r) \times(V-\min (A, r)) /(\max (A, r)-\min (A, r)) \\
& =\mathrm{V}(A, r) \times \mathrm{P}(A \leq v)
\end{aligned}
$$

- In general, if $\theta$ is $A$ op $v$ :
- op is some inequality comparison: $<>\leq \geq \neq$
- Estimate $\mathrm{V}\left(A, \sigma_{\theta}\right)=\mathrm{V}(A, r) \times \mathrm{P}(\theta)$


## Selection Against Subplans (3)

- If predicate $\theta$ forces $A$ to take on a set of values:
- SELECT * FROM schedule WHERE hour = 3 OR hour = 4;
- SELECT * FROM shapes WHERE color IN ('red', 'orange', 'yellow');
- $\mathrm{V}\left(A, \sigma_{\theta}\right)=$ number of values in the predicate
- Can compute $\min \left(A, \sigma_{\theta}\right), \max \left(A, \sigma_{\theta}\right)$ from these as well
- If none of these situations occur:
- Assume $V\left(A, \sigma_{\theta}\right), \min \left(A, \sigma_{\theta}\right), \max \left(A, \sigma_{\theta}\right)$ are independent of selection criteria!
- Set $\mathrm{V}\left(A, \sigma_{\theta}\right)$ to $\min \left(\mathrm{V}(A, r), n_{\sigma}\right)$
- \# of distinct values for $A$ is capped by $\#$ of rows produced by $\sigma$


## Join Costs

- Several important costs to estimate for joins
- Number of rows produced by the join operation
- Number of disk IOs performed by the join operation
- Second value is harder to estimate, primarily due to the buffer manager, but still critical to estimate
- Example: nested loop join (no optimizations)
- Worst case (unlikely): $b_{r}+n_{r} \times b_{s}$ block reads
- Best case (inner table fits in memory): $b_{r}+b_{s}$ reads
- Disk IO estimate is very approximate, and depends on the specific join implementation being used


## Join Costs (2)

- For now, focus on the number of rows produced
- Cartesian product: $r \times s$
- Every row in table $r$ is joined to every row in table $s$
- $n_{r \times s}=n_{r} \times n_{s}$
- Average tuple length $l_{r \times s}=l_{r}+l_{s}$
- Theta join: $r \bowtie_{\theta} s$
- Can model as $\sigma_{\theta}(r \times s)$; compute estimates as for $\sigma_{\theta}(\ldots)$
- Big problem: our cost estimates are most accurate when comparing attributes to constants!
- Join predicates usually compare attributes to attributes


## Join Costs (3)

- To compute proper join estimates, need to look at the attributes being compared
- For theta-join $r \bowtie_{r . A=S . A} s$ :
- If $r . A$ is a key for $r$ :
- Each tuple in $s$ will join with at most one tuple in $r$
- Estimate number of tuples in result $n_{r \propto s}=n_{s}$
- Similarly, if $s . A$ is a key for $s$ :
- Each tuple in $r$ will join with at most one tuple in $s$
- Estimate $n_{r 凶 s}=n_{r}$
- If both are keys for their respective tables:
- $n_{r \bowtie s}=\min \left(n_{r}, n_{s}\right)$


## Join Costs (4)

- For theta-join $r \bowtie_{r . A=S . A} s$ :
- If neither r.A nor $s . A$ is a key for its respective table:
- Assume that $A$ is uniformly distributed in both $r$ and $s$
- (Note: ignoring min/max stats for these estimates)
- Given a specific tuple $t_{r}$ in $r$, estimate that $n_{s} / \mathrm{V}(A, s)$ tuples in $s$ will join with that tuple
- $n_{s} \times$ probability that a given tuple $t_{s}$ in $s$ will have value $t_{r} A$
- Suggests that $n_{r 凶 s}=n_{r} \times n_{s} / V(A, s)$
- But, given a specific tuple $t_{s}$ in $s$, estimate $n_{r} / \mathrm{V}(A, r)$ tuples in $r$ will join with that tuple
- Suggests that $n_{r 凶 s}=n_{s} \times n_{r} / V(A, r)$


## Join Costs (5)

- For theta-join $r \bowtie_{r . A=S . A} S$ :
- Two estimates for number of rows produced:
- $n_{r \times s}=n_{r} \times n_{s} / \mathrm{V}(A, s)$
- $n_{r \times s}=n_{s} \times n_{r} / \mathrm{V}(A, r)$ (from perspective of tuples in $r$ )
(from perspective of tuples in $s$ )
- If $\mathrm{V}(A, r)<\mathrm{V}(A, s)$ :
- Expect that more tuples in $s$ will not join with any tuple in $r$
- Use estimate based on $r: n_{r \propto s}=n_{r} \times n_{s} / \mathrm{V}(A, s)$
- Similarly, if $\mathrm{V}(A, r)>\mathrm{V}(A, s)$, more tuples in $r$ will be left out
- If $\mathrm{V}(A, r) \neq \mathrm{V}(A, s)$, choose the larger of $\mathrm{V}(A, r), \mathrm{V}(A, s)$
- Estimate $n_{r \times s}=n_{r} \times n_{s} / \max (\mathrm{V}(A, r), \mathrm{V}(A, s))$


## Join Costs (6)

- Can extend these estimates to joins with multiple conjuncts
- For theta-join $r \bowtie_{r . A=S . A \wedge r . B=S . B} S$ :
- Check if $(r . A, r . B)$ or any proper subset is a key for $r$
- Check if ( $s . A, s . B$ ) or any proper subset is a key for $s$
- If so, compute estimates as before
- If attributes are not keys for $r$ or $s$ :
- Again, assume the conditions are independent of each other
- $\mathrm{P}(r . A=s . A \wedge r . B=s . B)=\mathrm{P}(r . A=s . A) \times \mathrm{P}(r . B=s . B)$

$$
=1 /(\max (\mathrm{V}(A, r), \mathrm{V}(A, s)) \times \max (\mathrm{V}(B, r), \mathrm{V}(B, s)))
$$

- $n_{r \propto s}=n_{r} \times n_{s} /(\max (\mathrm{V}(A, r), \mathrm{V}(A, s)) \times \max (\mathrm{V}(B, r), \mathrm{V}(B, s)))$


## Outer Join Costs

- Can use very simple estimates for outer joins
- Again, only using number of distinct values; not using min/max to further refine statistics
- Left outer join: $n_{r \rtimes s}=n_{r \bowtie s}+n_{r}$
- Right outer join: $n_{r \bowtie s}=n_{r \bowtie s}+n_{s}$
- Full outer join: $\quad n_{r æ s}=n_{r \bowtie s}+n_{r}+n_{s}$
- These estimates are almost certainly much higher than actual row-counts will be, but they are an upper bound
- ...and they are fast to compute.
- Could devise a better estimate, but really want to move to better stats (e.g. storing histograms) to make it worthwhile


## Other Plan Nodes

- Project: П...(r)
- $\Pi_{A}(r)$, where $A$ is a simple column-reference
- $\mathrm{n}_{\mathrm{\Pi}}=\mathrm{n}_{\mathrm{r}}$ (no duplicate-elimination in SQL)
- $\mathrm{V}\left(\mathrm{A}, \Pi_{\mathrm{A}}\right)=\mathrm{V}(\mathrm{A}, \mathrm{r})$
- Similarly, min/max don't change
- $\Pi_{\mathrm{E}}(\mathrm{r})$, where E is an expression possibly with functions
- Again, $\mathrm{n}_{\Pi}=\mathrm{n}_{\mathrm{r}}$
- For $V\left(E, \Pi_{E}\right) / \min \left(E, \Pi_{E}\right) / \max \left(E, \Pi_{E}\right)$, no idea! Either need to guess, or we need more knowledge about E .
- E.g. just guess $V\left(E, \Pi_{E}\right)=n_{\Pi}$


## Other Plan Nodes (2)

- Grouping/aggregation: ${ }_{\mathrm{G} 1, \mathrm{G} 2, \ldots} \mathcal{G}_{\mathrm{E} 1, \mathrm{E} 2, \ldots(\mathrm{r})}$
- Gi can be either column-references or expressions
- Ei can be simple aggregate function calls, or more advanced expressions involving aggregate functions
- SELECT SUM(CASE WHEN a < b THEN 1 ELSE 0 END) FROM t;
- SELECT MIN(a) + MAX(b) FROM t;
- For simple column-references in grouping attributes:
- $n_{G}=V(G 1, r) \times V(G 2, r) \times \ldots$
- $\mathrm{V}(\mathrm{G} 1, \mathrm{G})=\mathrm{V}(\mathrm{G} 1, \mathrm{r})$, etc.


## Other Plan Nodes (3)

- Grouping/aggregation: ${ }_{\mathrm{G} 1, \mathrm{G} 2, \ldots} \mathcal{G}_{\mathrm{E} 1, \mathrm{E} 2, \ldots(\mathrm{r})}$
- For simple column-references and simple aggregates:
- Guess COUNT(A), SUM(A), AVG(A) will produce different values for each group. e.g. $\operatorname{V}(\operatorname{COUNT}(A), \mathcal{G})=n_{G}$
- Can be a bit more clever with $\operatorname{MIN}(\mathrm{A})$ and $\operatorname{MAX}(\mathrm{A})$
- Could guess V(MIN(A), $\mathcal{G})=\mathrm{n}_{\mathcal{G}}$ as before
- Note that MIN(A)/MAX(A) will always select an existing value of A from input relation
- $A$ better guess: $\operatorname{V}(\operatorname{MIN}(A), \mathcal{G})=\min \left(V(A, r), n_{G}\right)$


## Summary - Plan Costing

- Plan costing is a very imprecise process
- Almost certainly inaccurate, except in very simple cases
- Hopefully estimates are "good enough" to guide plan selection
- (Most databases provide ways to give the optimizer hints about plan optimization)
- These estimates are simply one way of estimating costs
- Different assumptions, or different kinds of statistics, will produce different costing estimates
- Still, an essential part of query planning!
- Collecting useful table stats, then making reasonably accurate estimates from them, greatly improves DB query performance
- (Becomes very obvious when table stats are inaccurate)


## Equivalent Plans?

- Previously had this query:
- SELECT * FROM t1, t2 WHERE t1.a = t2.a AND t2.b > 5;

- How do we know these plans are actually equivalent?


## Equivalent Plans

- Two plans are equivalent if they produce the same results for every legal database instance
- A "legal" database instance satisfies all constraints
- Generally, the order of tuples is irrelevant
- If sorting is not specified on results, two equivalent plans may generate results in different orders
- Equivalence rules specify different forms of an expression that are equivalent
- Can prove that these rules hold for all legal databases
- Can use them to transform query plans into equivalent (but hopefully faster) plans


## Simple Equivalence Rules

- Cascade of $\sigma$ :

$$
\text { - } \sigma_{\theta 1 \wedge \theta 2}(\mathrm{E})=\sigma_{\theta 1}\left(\sigma_{\theta 2}(\mathrm{E})\right)
$$

- $\sigma$ is commutative:
- $\sigma_{\theta 1}\left(\sigma_{\theta 2}(E)\right)=\sigma_{\theta 2}\left(\sigma_{\theta 1}(E)\right)$
- Selections, Cartesian products, and theta-joins:
- $\sigma_{\theta}(\mathrm{E} 1 \times \mathrm{E} 2)=\mathrm{E} 1 \bowtie_{\theta} \mathrm{E} 2$
- $\sigma_{\theta 1}\left(\mathrm{E} 1 \bowtie_{\theta 2} \mathrm{E} 2\right)=\mathrm{E} 1 \bowtie_{\theta 1 \wedge \theta 2} \mathrm{E} 2$
- Theta-joins are commutative:
- E1 $\bowtie_{\theta}$ E2 = E2 $\bowtie_{\theta}$ E1


## Theta Join Equivalence Rules

- Natural joins are associative:
- $(\mathrm{E} 1 \bowtie \mathrm{E} 2) \bowtie \mathrm{E} 3=\mathrm{E} 1 \bowtie(\mathrm{E} 2 \bowtie \mathrm{E} 3)$
- Theta-joins are also associative, but it's a bit trickier:
- $\left(E 1 \bowtie_{\theta 1} \mathrm{E} 2\right) \bowtie_{\theta 2 \wedge \theta 3} \mathrm{E} 3=\mathrm{E} 1 \bowtie_{\theta 1 \wedge \theta 3}\left(\mathrm{E} 2 \bowtie_{\theta 2} \mathrm{E} 3\right)$
- $\theta 1$ only refers to attributes in E1 and/or E2
- $\theta 2$ only refers to attributes in E2 and/or E3
- $\theta 3$ only refers to attributes in E1 and/or E3
- Any of these conditions might also simply be true


## Theta Join Equivalence Rules (2)

- Can sometimes distribute selects over theta-joins:
- $\sigma_{\theta 1}\left(\mathrm{E} 1 \bowtie_{\theta} \mathrm{E} 2\right)=\sigma_{\theta 1}(\mathrm{E} 1) \bowtie_{\theta} \mathrm{E} 2$
- $\theta 1$ only refers to attributes in E1
- $\sigma_{\theta 1 \wedge \theta 2}\left(\mathrm{E} 1 \bowtie_{\theta} \mathrm{E} 2\right)=\sigma_{\theta 1}(\mathrm{E} 1) \bowtie_{\theta} \sigma_{\theta 2}(\mathrm{E} 2)$
- $\theta 1$ only refers to attributes in E1
- $\theta 2$ only refers to attributes in E2


## Equivalence Rules

- Many other equivalence rules besides these
- Cover grouping, projects, outer joins, set operations, duplicate elimination, sorting, etc.
- Grouping: $\sigma_{\theta}\left({ }_{A} \mathcal{G}_{\mathrm{F}}(\mathrm{E})\right.$ ) is equivalent to ${ }_{\mathrm{A}} \mathcal{G}_{\mathrm{F}}\left(\sigma_{\theta}(\mathrm{E})\right)$
- ...as long as $\theta$ only involves attributes in A !
- Outer joins: $\sigma_{\theta}(\mathrm{E} 1 \not \Perp \mathrm{E} 2)$ is equivalent to $\sigma_{\theta}(\mathrm{E} 1) \not \searrow \mathrm{E} 2$
- $\theta$ only involves attributes in E1


## Equivalence Rules

- Equivalence rules allow us to transform plans, and know the results will not change:



## Outer Join Transformations

- Need to be very careful transforming outer joins:
- Obviously correct equivalences for natural joins / theta joins don't necessarily hold for outer joins!
- Is $\sigma_{\theta}(\mathrm{E} 1 \searrow \mathrm{E} 2)$ equivalent to $\mathrm{E} 1 \searrow \sigma_{\theta}(\mathrm{E} 2)$ ?
- $\theta$ only uses attributes in E2
- These are not equivalent. Example:
- $r(A, B)$ with one row $\{(1,2)\}$
- $s(B, C)$ with one row $\{(2,3)\}$
- $\theta$ is $\mathrm{C}=1$
- $\sigma_{\mathrm{C}=1}(\mathrm{r} \not \searrow \mathrm{s})=\{ \}$ (empty relation), but $\mathrm{r} \ngtr \sigma_{\mathrm{C}=1}(\mathrm{~s})=\{(1,2, n u l)\}$


## Outer Join Transformations (2)

- Need to be very careful transforming outer joins:
- Obviously correct equivalences for natural joins / theta joins don't necessarily hold for outer joins!
- Is $(E 1 \searrow E 2) \not \Perp E 3$ equivalent to $E 1 \searrow \rtimes(E 2 \neg E 3)$ ?
- These are not equivalent. Example:
- $r(A, B)$ with one row $\{(1,2)\}$
- $s(A, C)$ with one row $\{(2,3)\}$
- $\mathrm{t}(\mathrm{A}, \mathrm{D})$ with one row $\{(1,4)\}$
- $(\mathrm{r} \rtimes \mathrm{s}) \nexists \mathrm{t}=\{(1,2, n u l l)\} \ngtr \mathrm{t}=\{(1,2$, null, 4$)\}$
- $\mathrm{r} \ngtr(\mathrm{s} \nexists \mathrm{t})=\mathrm{r} \nexists\{(2,3$, null $)\}=\{(1,2$, null, null $)\}$


## Query Plan Optimization

- Generally understand how to map SQL queries to plans
- Ignoring subqueries in SELECT and WHERE clauses for the time being...
- Understand how to implement basic plan nodes
- Still a lot of optimizations to cover though...
- A query can be evaluated by many different plans...
- How do we find an optimal plan to evaluate a query?
- Many different approaches
- All depend on equivalence rules to guide generation of equivalent plans

