Geometric Modeling: CS 175

Peter Schröder
& Ke Wang (TA)

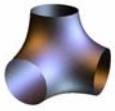
T/Tr 14.30-16.00 Firestone 308

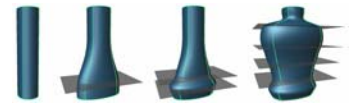
Geometric Modeling

Representation of geometric shapes in the computer

- mathematical foundations
- algorithms







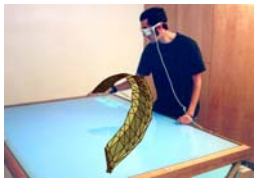
GEOMETRIC MODELING

Goals

- model geometry of real world objects
  - car bodies, cells, turbulent flow, etc.
- model geometry of abstract objects
  - approximation theory
  - smoothness spaces

GEOMETRIC MODELING

Model building



- interactive modeling
- data matching
  - build shapes from measurement
  - interpolation/approximation

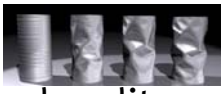
Compute with it

- FEM simulation, evolving geometry

Applications

Relevant for:

- engineering:
  - CAGD, simulation, visualization
- entertainment:



  - animation, games, virtual reality
- science:

  - medicine, biology

Issues

Time and space efficiency
- ideally linear complexity

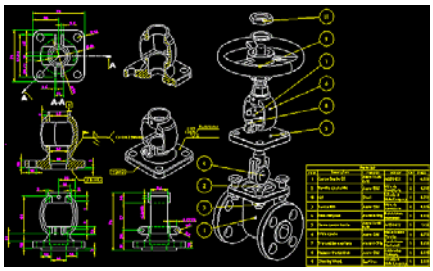
Precision and consistency
- geometric and topologic errors

Flexibility
- large class of possible shapes

INDUSTRY

Manufacture from many parts

- precision
- consistency



ENTERTAINMENT

Shapes are not static

- need good controls (intuitive)

Animation

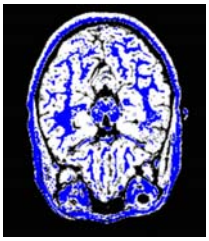
- flexibility, precision

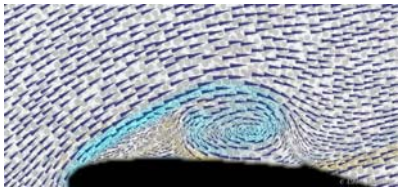
Games

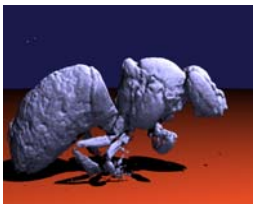
- speed, real-time

Visualization

Space efficiency
- huge data sets
- surface extraction
- volumetric data







Representing Shapes

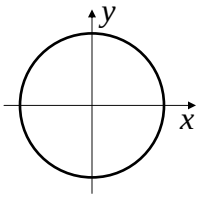
Explicit: $y = f(x)$

$y = -(x^2 - 1)^{1/2}$ $\quad -1 \leq x \leq 1$

$y = (x^2 - 1)^{1/2}$

Implicit: $F(x, y) = 0$

$x^2 + y^2 - 1 = 0$

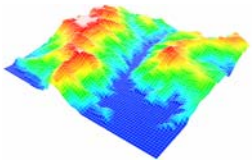


Parametric: $(x(t), y(t))$

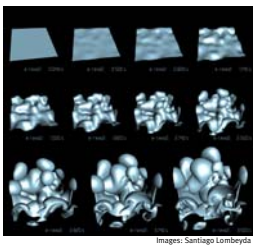
$x = \cos(t) \quad y = \sin(t) \quad -\pi < t \leq \pi$

SURFACES

Explicit
- terrains

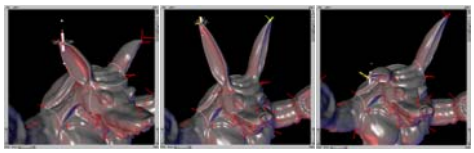


Implicit
- changing topology
- volumetric



Images: Santiago Lombeyda

Parametric
- flexible
- easy to transform and discretize

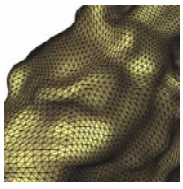


COMPARISON

Explicit: not flexible enough

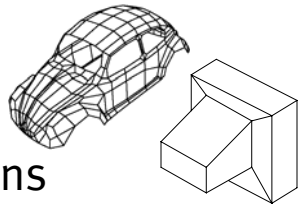
Implicit
- smooth shapes (“blobbies”)

Parametric
- very flexible
- easy to discretize



Polygonal Models

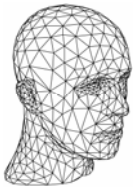
Polygon - universal primitive
- direct in hardware



Polygonal mesh
- collection of polygons

Quad mesh:

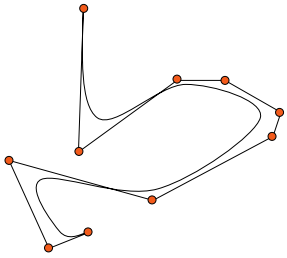
$$\{(p_{n_1}, p_{n_2}, p_{n_3}, p_{n_4}) | n = 1, \ldots, N\}$$



But – not smooth

Control Meshes

Polygonal mesh – controls shape of a smooth model

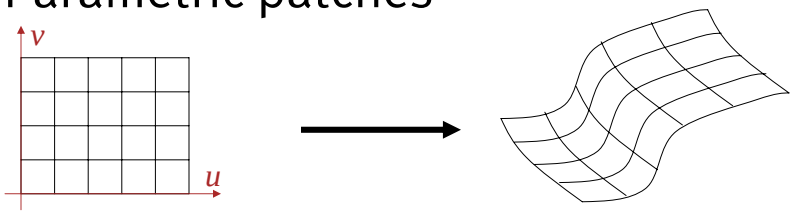


User drags control vertices to change smooth shape

Geometric Primitives

Polygonal meshes

Parametric patches



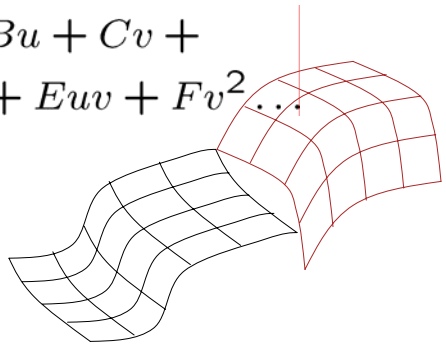
Subdivision Meshes

Parametric Patches

Polynomials

$$s^l(u,v) = A + Bu + Cv + Du^2 + Euv + Fv^2 \ldots$$

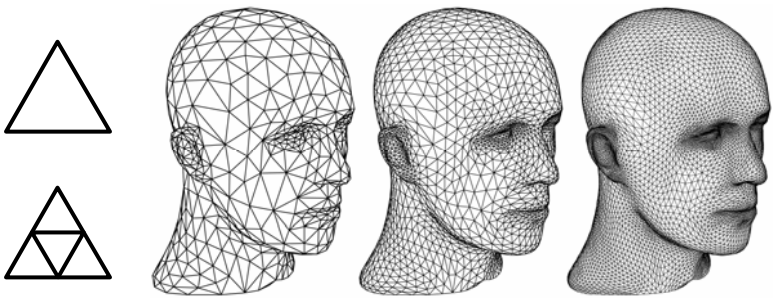
$$s^r(u,v) = \ldots$$



How to match the two patches?

SUBDIVISION

Smooth surfaces as the limit of a sequence of refinements



CS 175

Classic methods

- piecewise polynomials
  - curves and surfaces
  - splines, NURBS

State of the art methods

- subdivision curves and surfaces

Rules of the Game

- Homeworks
  - theory as well as programming
  - no midterm, no final
- Project
  - latest research and maybe beyond
    - theory/implementation
    - individual or group

Rules of the Game

- Extensions
  - can only be granted by professor
  - late penalty
    - 20% up to 24 hours late
    - 40% up to 48 hours late
    - don't bother after that