# Polynomial Surfaces

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## Surfaces

Non-tensor product

$$F(u,v) = \sum_{i+j+k=n} p_{ijk} \binom{n}{ijk} u^i v^j (1-u-v)^k$$

- example: quadratic in two variables

$$F(u,v) = a_{00} + a_{10}u + a_{01}v + a_{20}u^2 + a_{11}uv + a_{02}v^2$$

- Bezier functions for triangles
- B-patches? very hard!

## Surfaces

Tensor product

- 2 parameter directions which decouple $F(u,v) = F_u(u)F_v(v)$
- example: bi-cubic patches

$$
\begin{aligned}
F(u,v) &= \sum_{i,j=0}^{3} p_{ij} B_i^3(u) B_j^3(v) \\
&= (a_0 + a_1 u + a_2 u^2 + a_3 u^3)(b_0 + b_1 v + b_2 v^2 + b_3 v^3)
\end{aligned}
$$



## Tensor Product Surf.

Polarform

- reduces to 1D setting

$$F(u,v) = F_u(u)F_v(v) = f_u(uuu)f_v(vvv) = f(uuu;vvv)$$

$$F(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} f(u_1^{n-i} u_2^{i}; v_1^{m-j} v_2^{j}) B_i^n(u) B_j^m(v)$$

- control points of tensor product surface are outerproduct of control points for 1D components

# TENSOR PRODUCT

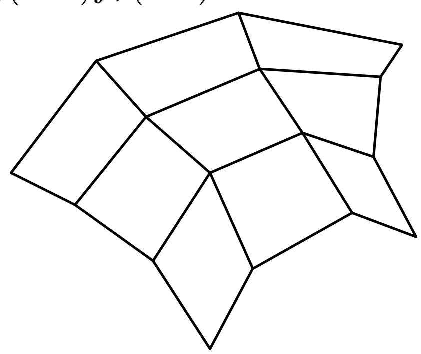
## Control points

$$F(u,v) = f_u(uuu)f_v(vvv)$$

u →

v ↓

| | a | b | c | d |
|---|---|---|---|---|
| e | ea | eb | ec | ed |
| f | fa | fb | fc | fd |
| g | ga | gb | gc | gd |
| h | ha | hb | hc | hd |



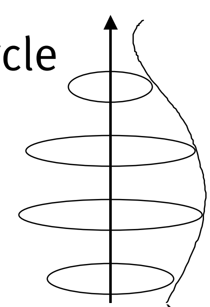


# SURFACES OF REVOLUTION

## Profiles and axes

- modulate diameter of circle

$$s(u) = \begin{pmatrix} B_x^s(u)/B_w^s(u) \\ B_y^s(u)/B_w^s(u) \end{pmatrix}$$



$$T(u,v) = \begin{pmatrix} r(v)s(u) \\ a(v) \end{pmatrix} = \begin{pmatrix} B_x^r(v)/B_w^r(v)s(u) \\ B_z^r(v)/B_w^r(v) \end{pmatrix}$$