An Architectural View of Game Theoretic Control

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ABSTRACT

Game-theoretic control is a promising new approach for distributed resource allocation. In this paper, we describe how game-theoretic control can be viewed as having an intrinsic layered architecture, which provides a modularization that simplifies the control design. We illustrate this architectural view by presenting details about one particular instantiation using potential games as an interface. This example serves to highlight the strengths and limitations of the proposed architecture while also illustrating the relationship between game-theoretic control and other existing approaches to distributed resource allocation.

1. INTRODUCTION

Resource allocation is a fundamental problem that arises in nearly all computer systems, and increasingly it is a problem that needs to be solved in a distributed, decentralized manner, e.g., power control and frequency selection problems in wireless networks and coverage problems in sensor networks. Resultantly, there is a large and growing literature that focuses on developing distributed resource allocation protocols. This is an extremely diverse literature where protocols are designed using a wide variety of tools, e.g., distributed optimization [28, 44], distributed control [21, 32], physics-inspired control (e.g. Gibbs-sampler-based control) [17, 27], and game-theoretic control [2, 4, 42].

In this paper, we focus on game-theoretic control, which is a promising new approach for distributed resource allocation. The game-theoretic approach involves modeling the interactions of agents within a noncooperative game where the agents are 'self-interested'. This is motivated by the fact that the underlying decision making architecture in economic systems is identical to the desired decision making architecture in distributed engineering systems, i.e., local decisions based on local information where the global behavior emerges from a compilation of these local decisions. This parallel makes it possible to utilize the broad set of economic/game-theoretic tools in distributed control. However, a key distinction between game theory for economic systems and game theory for engineering systems is that decision makers are inherited in economic systems while decisions makers are designed in engineering systems. This difference means that using game-theoretic tools for distributed control requires a new perspective on the economic literature.

Applying game-theoretic control requires requires specifying decision makers, their respective choices, their objective/utility functions, and the learning rules for the agents. In this paper, we focus on two of these components: (i) the design of the agents' utility functions, i.e., *utility design*, and (ii) the design of the distributed learning rules for the agents, i.e., *learning design*. The goal is to design the utility functions and the learning rules so that the emergent global behavior is desirable. There are wide-ranging advantages to the game-theoretic approach including robustness to failures and environmental disturbances, minimal communication requirements, and improved scalability.

Game-theoretic resource allocation designs are increasingly popular in a variety of wireless and sensor network applications, e.g., channel access control in wireless networks [5, 17], coverage problems in sensor networks [8], and power control in both [3, 7, 14]. A comprehensive survey of applications can be found in [4]. However, nearly all of these designs are highly application-specific, with both the utility and learning designs crafted carefully for the specific setting. There have been only a few papers that focus on general designs and even these papers tend to focus on only one aspect of the design – either the utility design, e.g., [25, 26], or the learning design, e.g., [22, 23, 37].

Our contribution in this paper is to present an 'architectural' view of game-theoretic control as a whole. We describe, at a high-level, our proposed architecture for game-theoretic control in Section 2. Then, to make the ideas more concrete, in Section 3 we present details about one particular instantiation of the architecture, based on potential games, which aligns with a number of current approaches. Additionally, we provide some new results highlighting how two other existing approaches for distributed system design (distributed constraint optimization and Gibbs-sampler-based control) can be viewed as instances of designs in this architecture. Finally, in Section 4 we a discuss a number of important research directions outside of the potential games framework that are suggested by the architectural view of game-theoretic control.

2. A LAYERED ARCHITECTURE

Game-theoretic control has two key design tasks: utility design and learning design. Each of these tasks is complex and subject to application-specific constraints. Further, these two designs must be done in conjunction with one another in order to ensure desirable global behavior. Resultantly, many designers have carefully performed application-specific *co-design* of the utility functions and the learning rule, e.g., [4, 33, 34, 36], which is typically a difficult task.

The central question that this paper explores is how to achieve a *modularization/decoupling* of utility and learning design. Such a decoupling would allow for the development of a rich set of utility and learning designs from which a design can be chosen 'off the shelf' according to the requirements of the resource allocation problem being considered.

It turns out that game theoretic control can naturally be viewed within an 'hourglass' architecture, which provides the desired modularity. An hourglass architecture is a type of layered architecture where within the highest and lowest layers there is a large diversity of available designs, but near the middle, or waist, the design is highly constrained. The most famous example of such an 'hourglass' architecture is the IP network stack [20, 30, 45]; however, this architecture is quite

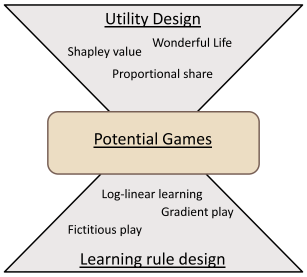

Figure 1: An illustration of the 'hourglass' architecture using potential games as the interface.

common in computer systems and has also been observed in wide-ranging areas such as biology [12].

In the context of game-theoretic control, this architecture allows for a diverse set of possible utility and learning designs; but simplifies design by enforcing a particular structure on the games that can result from a utility design and on the games for which a learning design will ensure desirable global behavior. This enforced structure then serves as the interface between utility design and learning design; thus providing the desired modularity.

In Section 3, we discuss a concrete example of this constrained interface, the class of *potential games*. (See Figure 1 for an illustration.) This interface requires utility designs to guarantee that the resulting game is a potential game and requires learning rules to guarantee to provide desirable behavior when run on a potential game. Thus, requiring the structure of potential games enforces additional *constraints* on utility and learning design, but also *deconstrains* at a higher level by allowing modularization. Though this layered architecture was not explicitly used in prior work, the modularization provided by potential games underlies many successful examples of game-theoretic control [7, 16, 19, 25]. The change in perspective provided by this architectural view is not simply superficial; it highlights that the utility and learning designs in these papers can be 'mixed and matched' while still obtaining the same performance.

Though we focus on potential games in much of this paper, it is important to remember that they are not the only choice for the interface – we discuss moving beyond potential games in Section 4.

3. LAYERING VIA POTENTIAL GAMES

We now illustrate how using potential games as an interface provides modularization of utility and learning design. We focus on potential games because many recent applications of game-theoretic control have relied on potential games, e.g., [7, 16, 19, 25]. A key reason that potential games are a powerful choice for the interface is that they are a highly studied class of games in the economics literature, e.g., [13, 29, 35, 43, 47] and so there is a large literature of results that can be used in the context of game-theoretic control for both utility and learning design.

In this section, we highlight the variety of utility and learning designs that have been adapted from the economics literature and can be used interchangeably 'off the shelf', greatly simplifying the task of design. However, we also illustrate that layering via potential games has some limitations, which highlights the need to consider other interfaces as well. Finally, we illustrate the relationship between layering with potential games and two other distributed design approaches. In order to illustrate these issues formally, we first define a simple resource allocation model.

3.1 Preliminaries

3.1.1 A model for resource allocation

Consider a set of distributed agents $N = \{1, \dots, n\}$ and a set of resources $R = \{r_1, \dots, r_m\}$ that are to be shared by the agents. Each agent $i \in N$ is capable of selecting potentially multiple resources in R; therefore, we say that agent i has action set $\mathcal{A}_i \subseteq 2^R$. An allocation, or an action profile, is represented by a tuple $a = (a_1, a_2, ..., a_n) \in \mathcal{A}$ where the set of possible allocations is denoted by $\mathcal{A} = \mathcal{A}_1 \times \ldots \times \mathcal{A}_n$. We will frequently denote an allocation a as (a_i, a_{-i}) where $a_{-i} \in \mathcal{A}_{-i} = \prod_{j \neq i} \mathcal{A}_j$ denotes the allocation of all agents except agent i.

The social welfare function $W(a)$ captures the global valuation of the agent allocation. In general, a resource allocation design seeks to find an allocation that optimizes the global welfare. In this work, we assume $W(a)$ is *linearly separable* across resources, i.e., $W(a) = \sum_{r \in R} W_r(\{a\}_r)$ where $\{a\}_r = \{i \in N : r \in a_i\}$ is the set of agents that are allocated to resource r in a and $W_r : 2^N \to \mathbb{R}_+$ is the local welfare function for resource r. Hence, the welfare generated at a particular resource depends only on which agents are allocated to that resource. Further, we restrict our attention to *submodular* welfare functions, i.e., for each resource $r \in R$ and any player sets $X \subseteq Y \subseteq N$,

$$W_r(X) + W_r(Y) \geq W_r(X \cup Y) + W_r(X \cap Y).$$

A variety of resource allocation problems such as power control and coverage problems in sensor networks [7, 25], wireless access point assignment and frequency selection [17], and influence maximization [18] all have linearly separable, submodular welfare functions.

3.1.2 Resource allocation games

Our goal is to utilize game theory to obtain distributed solutions to such resource allocation problems. This goal requires modeling the interactions of the agents in a noncooperative game theoretic environment where the agents act in a self-interested fashion. While we inherit the players N, the welfare function W, and the action sets $\{\mathcal{A}_i\}_{i \in N}$, we are left to design a utility function for each player of the form $U_i : \mathcal{A} \to \mathbb{R}$. A resource allocation game G is then defined by the tuple $G = (N, \{\mathcal{A}_i\}, \{U_i\}, W)$.

In general, a system designer has free reign in the design of utility functions; however, layering via potential games requires that the utility functions lead to a potential game. Formally, a game is called a *potential game*, if there exists a potential function $\Phi : \mathcal{A} \to \mathbb{R}$ such that $\forall\ i$, $\forall\ a_{-i} \in \mathcal{A}_{-i}$, and $\forall\ a_i, a_i' \in \mathcal{A}_i$:

$$\Phi(a_i, a_{-i}) - \Phi(a_i', a_{-i}) = U_i(a_i, a_{-i}) - U_i(a_i', a_{-i}).$$

Potential games possess several nice properties that can be utilized in distributed control. One such property is the guaranteed existence of a *pure Nash equilibrium* in any potential game. A *(pure Nash) equilibrium* is an action profile $a^* \in \mathcal{A}$ such that for each player i, $U_i(a_i^*, a_{-i}^*) = \max_{a_i \in \mathcal{A}_i} U_i(a_i, a_{-i}^*)$. In a distributed system, a pure Nash equilibrium takes on the role of a stable operating point.

To discuss the efficiency of games we use the notions of Price of Anarchy (PoA) and Price of Stability (PoS) [31], which compare the welfare of the set of equilibria to the globally optimal welfare. Let $\mathcal{G}$ denote a set of games and $\mathcal{S}(G)$ denote the set of equilibria for a game G. Then

$$PoA(\mathcal{G}) = \inf_{G \in \mathcal{G}} \left(\min_{a^* \in \mathcal{S}(G)} \frac{W(a^*)}{\max_{a \in \mathcal{A}} W(a)} \right)$$

$$PoS(\mathcal{G}) = \inf_{G \in \mathcal{G}} \left(\max_{a^* \in \mathcal{S}(G)} \frac{W(a^*)}{\max_{a \in \mathcal{A}} W(a)} \right)$$

So, the PoA measures the worst-case efficiency of any equilibrium while the PoS measures the worst-case efficiency of the best equilibrium across all games.

3.2 Utility Design

There are several desirable properties that a global planner must consider when designing utility functions. These properties include the following conditions on utility functions: (i) computable using only local information, (ii) guarantee the existence of an equilibrium, (iii) computable in polynomial time, (iv) guarantee PoS=1, (v) guarantee a PoA close to 1, and (vi) guarantee that utilities are budget-balanced, i.e., the sum of the utilities of all the players is equal to the social welfare for every action profile. In the context of the considered resource allocation games, we want to design utility functions that are *linearly separable*, i.e., that satisfy

$$U_i(a_i, a_{-i}) = \sum_{r \in a_i} f_r(i, \{a\}_r),$$

where $f_r : N \times 2^N \to \mathbb{R}$, i.e., a player's utility is only dependent on the resources the players choose and the other players that choose the same resources.

Recent work [25] has observed that the task of utility design is strongly related to the cost sharing literature in economics. Here, we present two promising utility designs that have emerged, though it is important to point out that there are a variety of other possible choices, e.g., [10, 25].

(i) *Wonderful life utility design (WLU)* [46]: This design defines the utility of each player i as their marginal contribution to the social welfare,

$$\begin{aligned} U_i(a_i, a_{-i}) &= W(a_i, a_{-i}) - W(\emptyset, a_{-i}) \\ &= \sum_{r \in a_i} W_r(\{a\}_r) - W_r(\{a\}_r \setminus i). \end{aligned}$$

Note that this is linearly separable. Additionally, it has been shown in [25] that WLU leads to a potential game with $\Phi = W$ [46] and that $PoS = 1$ and $PoA = 1/2$ for submodular resource allocation games. But, WLU is not budget-balanced.

(ii) *Shapley value utility design (SVU)* [40]: This design defines the utility of each player i as their Shapley value to the social welfare,

$$U_i(a_i, a_{-i}) = \sum_{r \in a_i} Sh_i(\{a\}_r; W_r)$$

where $Sh_i(\{a\}_r; W_r)$, the Shapley value of player i at resource r, is

$$\sum_{S \subseteq \{a\}_r : i \in S} \frac{(|\{a\}_r| - 2)!(|S| - 1)!}{|\{a\}_r|!} (W_r(S) - W_r(S \setminus \{i\})).$$

Notice that this utility design is also linearly separable. Furthermore, SVU leads to a potential game [43], is budget-balanced, and guarantees a $PoA = PoS = 1/2$ for submodular resource allocation games [25]. However, in general, SVU is not polynomial-time computable [11]. Note that there is also a 'weighted' SVU [39] that has the same properties.

3.3 Learning design

As with utility design, there are several desirable properties that a global planner must consider when designing distributed learning rules. These properties include (i) the asymptotic global behavior, (ii) equilibrium selection, (iii) informational dependencies and (iv) convergence rates.

Learning design takes on the form of a one-shot repeated game where at each time $t \in \{0, 1, 2, ...\}$ each player $i \in N$ simultaneously chooses an action $a_i(t) \in \mathcal{A}_i$ according to probability distribution $p_i(t)$ and receives a utility $U_i(a(t))$ where $a(t) = (a_1(t), ..., a_n(t))$. We refer to $p_i(t)$ as the *strategy* of player i at time t and denote the probability that player i will play action a_i at time t by $p_i^{a_i}(t)$. A players strategy at time t can rely only on actions (and their corresponding utilities) from times $\{0, 1, 2, ..., t-1\}$.

Here, we present two promising learning designs that have emerged, though it is important to point out that there are a variety of other possible choices, e.g., [15, 37, 38, 47].

(i) *Joint Strategy Fictitious Play (JSFP)* [23]: JSFP requires that each agent maintains a hypothetical payoff for each action a_i of the form

$$V_i^{a_i}(t) = \sum_{\tau=0}^{t-1} \frac{1}{t} U_i(a_i, a_{-i}(\tau)).$$

Note that this hypothetical payoff can be computed recursively and that a player only needs access to the payoff for alternative actions at each time step. Using this hypothetical payoff, the strategy of player i at time t is of the form

$$p_i^{a_i(t-1)}(t) = \epsilon, \quad p_i^{a_i^*}(t) = 1 - \epsilon$$

where $a_i^* \in \arg\max_{a_i \in \mathcal{A}_i} V_i^{a_i}(t)$ and $\epsilon > 0$ is referred to as 'inertia'. For any potential game, if all players adhere to this strategy, then the global behavior will converge almost surely to a pure Nash equilibrium [23].

(ii) *Log-linear learning* [6, 24]: In many settings, it is desirable converge to a specific Nash equilibrium, rather than just any equilibrium. This is termed 'equilibrium selection' and is not provided by JSFP. Log-linear learning provides equilibrium selection, but it also requires more structure on the learning environment. Under log-linear learning, at each time t, one agent $i \in N$ is randomly selected to update its action while all other agents are required keep their actions fixed, i.e., $a_{-i}(t) = a_{-i}(t-1)$. Player i plays a strategy at time t of the form

$$p_i^{a_i}(t) = \frac{e^{\frac{1}{T} U_i(a_i, a_{-i}(t-1))}}{\sum_{a_i' \in \mathcal{A}_i} e^{\frac{1}{T} U_i(a_i', a_{-i}(t-1))}}$$

where $T \geq 0$ is a temperature coefficient. For any potential game, if all players adhere to this mechanism, then the global behavior obeys an ergodic Markov chain with a unique stationary distribution given by:

$$\mu^a = \frac{e^{\frac{1}{T}\Phi(a)}}{\sum_{a' \in \mathcal{A}} e^{\frac{1}{T}\Phi(a')}}.$$

As the process cools (anneals), i.e., $T \to 0^+$, all the weight of the stationary distribution falls on the action profiles that maximize the potential function, thus providing equilibrium selection [24]. Often, e.g., for WLU where $\Phi = W$, this guarantees that the global performance achieves the bound set by the PoS, highlighting the importance of the PoS as a measure for efficiency.

3.4 Limitations

To this point we have highlighted that using potential games as an interface provides modularization and diverse options for utility and learning design. However, recent research [26] has identified that there are also fundamental limitations of layering via potential games. In particular, it is not possible for a utility design to achieve all of the properties one might desire.

For example, there are conflicts between maintaining a $PoS = 1$ and being budget-balanced. Specifically, a limitation of layering via potential games is that *any linearly* *separable, budget-balanced utility design that guarantees the existence of an equilibrium for all resource allocation games has* $PoS \leq 1/2$ [26]. WLU and SVU provide an illustration of this limitation: SVU is linearly separable and budget-balanced but has $PoS = 1/2$ while WLU is linearly separable and has $PoS = 1$ but is not budget-balanced.

Notice that this limitation focuses on budget-balanced utility design. While being local and having a PoS close to one are nearly always desirable, not all applications require that utilities be budget-balanced. Thus, in many cases this limitation may not be relevant. However, limitations of potential games have only begun to be studied, and there are likely more severe ones yet to be discovered.

3.5 Relationships to other approaches

In this section we highlight some interesting connections between layering via potential games and other distributed system design tools. We show formally that two other tools (distributed constraint optimization and Gibbs-sampler-based control) can be viewed as instances of layering via potential games (though they do not fall into our simple illustrative model of resource allocation games). This relationship highlights that in retrospect one could have used the layered architecture presented in this paper to design Gibbs-sampler based control modularly via 'off the shelf' utility and learning designs. Further, it highlights that other utility and learning designs can easily be swapped into these designs.

3.5.1 Gibbs-sampler-based control

Gibbs-sampler-based control [17, 27] is a popular physics-inspired approach for wireless protocol design. To introduce Gibbs-sampler based control we adapt the description of [17]. We represent a distributed system by an undirected graph $G(N, E)$ with $|N| = n$. Each node stores a state variable from a finite state space S. The state of the graph is $s = (s_1, \ldots, s_n)$. An energy function $\mathcal{E} : S^n \to \mathbb{R}$ represents the global cost of the system as a function of its state. The objective is to find a state of minimum energy. The Gibbs sampler approach provides an efficient solution for this problem, if $\mathcal{E}(s)$ is of the form

$$\mathcal{E}(s) = \sum_k \sum_{M \in C_k} V(M)$$

where C_k is the set of all cliques of order k, and $V : 2^N \to \mathbb{R}_+$ is defined such that $V(M)$ depends only on the states of nodes in M, and is zero if M is not a clique. We then define the *local energy* of a node $i \in N$ to be the sum of those terms in $\mathcal{E}(s)$ that involve s_i,

$$\mathcal{E}_i(s_i, (s_j)_{j \neq i}) = \sum_k \sum_{M \in C_k : i \in M} V(M)$$

The Gibbs measure associated with an energy function $\mathcal{E}$ and temperature $T > 0$ is defined as the following probability distribution on the states of the graph,

$$\pi(s) = e^{-\frac{\mathcal{E}(s)}{T}} \Big/ \left(\sum_{s' \in S^n} e^{-\frac{\mathcal{E}(s')}{T}} \right) \qquad (1)$$

The *Gibbs sampler* is an iterative procedure where during each step, each node i, given the states of all other nodes, samples its new state from the following distribution on S,

$$\mu(s_i) = e^{-\frac{\mathcal{E}_i(s_i, (s_j)_{j \neq i})}{T}} \Big/ \left(\sum_{s'_i \in S} e^{-\frac{\mathcal{E}_i(s'_i, (s_j)_{j \neq i})}{T}} \right) \;, \; s_i \in S$$

When T is fixed, the Gibbs sampler converges to a steady state that is distributed according to (1). Finally, for convergence to the global minimum of the energy function, we use the 'annealed' Gibbs sampler, which adds a small decrease of T to this algorithm at every step. When this decrease with time $t > 0$ is proportional to $1/\log(1+t)$, the system converges to a set of states of minimal global energy.

The above description of Gibbs-sampler-based control already highlights the connection with potential games – it is equivalent to using WLU in combination with log-linear learning. It is immediate to see that the Gibbs sampler is the same as the log-linear learning algorithm described earlier. To show that the utility design is equivalent to WLU, we construct a game as follows. Consider N to be the set of players, the common state space to be their action spaces, and the negative of the global energy function to be the social welfare function. Denote by $C_k(H)$, the set of cliques of order k in graph H. The WLU design is:

$$\begin{aligned} U_i(s) &= W(s) - W(\emptyset, s_{-i}) \\ &= -\mathcal{E}(s) + \mathcal{E}(\emptyset, s_{-i}) \\ &= -\sum_k \sum_{M \in C_k(G)} V(M) + \sum_k \sum_{M \in C_k(G - \{i\})} V(M) \end{aligned}$$

Since $M \in C_k(G - \{i\}) \iff M \in C_k(G)$ and $i \notin M$, all these terms cancel out, leaving only the terms in $-\mathcal{E}(s)$ for which $i \in M$. Hence,

$$U_i(s) = -\sum_k \sum_{M \in C_k : i \in M} V(M) = -\mathcal{E}_i(s)$$

3.5.2 Distributed constraint optimization

A constraint optimization problem is specified by a set of variables $N = \{1, \ldots, n\}$, each of which takes a value s_i from a finite state space S, a set of constraints $C = \{c_1, c_2, \ldots, c_m\}$, and a global objective function $W : S^n \to \mathbb{R}$, that encodes the relative desirability of each possible state s of the system in S^n. A constraint $c = \langle N_c, R_c \rangle$ is specified by the set of variables $N_c \subseteq N$ over which it is defined, and a relation $R_c \subset S^{|N_c|}$ between those variables. A function $U_c(s_c)$ specifies the reward for satisfying constraint c, where s_c is the configuration of the states of the variables in N_c. The global objective function is typically written as $W(s) = \sum_{c \in C} U_c(s_c)$. The problem is to find a global maximizer of W. Given this, a 'distributed' constraint optimization problem (DCOP) is produced when a set of autonomous agents each independently control the state of a variable.

In [9], the authors show that DCOPs can be viewed in the context of potential games, thus allowing any of the learning designs described earlier to be applied. Here, we highlight that this 'DCOP game' corresponds specifically to choosing a WLU design. Consider the autonomous agents of the DCOP to be the players, the common state space to be their action sets, and the global objective function to be the social welfare function. Denote by $C(M)$ the set of constraints involving any of the variables in the set $M \subseteq N$. WLU then gives:

$$\begin{aligned} U_i(s) &= W(s) - W(\emptyset, s_{-i}) \\ &= \sum_{c \in C(N)} U_c(s_c) - \sum_{c \in C(N \setminus \{i\})} U_c(s_c) \\ &= \sum_{c \in C(\{i\})} U_c(s_c) \end{aligned}$$

which is exactly the utility function suggested for the DCOP game in [9]. The last step follows by observing that when variable i is not part of the DCOP, all constraints not containing i in the original DCOP are not affected.

4. MOVING BEYOND POTENTIAL GAMES

To this point, we have discussed a concrete example of the game-theoretic control architecture – using potential games as an interface between utility and learning design. However, potential games are only one, very restrictive class of games, and research has begun to uncover limitations of layering via potential games (see Section 3.4).

Thus, as we move forward, it is important to consider other options for the interface. Recent research is beginning to consider a variety of other classes of games as the basis for game-theoretic control. For example, [26] suggests 'state-based potential games', which are a limited form of Markov games, as a way to overcome the limitation of potential games described in Section 3.4. Other examples include [41], which proposes using conjectural equilibria in the context of multi-user power control, and [1] which proposes using oblivious equilibria in the context of large stochastic games. However, as yet, there is little understanding of the strengths and limitations of designs using these new classes of games. For example, which classes of games provide modularity when used as an interface? What is gained by broadening the interface from potential games to other classes? Is there a penalty for broadening the interface, e.g. slower convergence rates for learning?

Our hope is that the identification of an architectural view of game-theoretic control in this paper can help formalize and motivate these important directions for the field. We propose that a better understanding of the strengths and weaknesses of differing interfaces will provide useful insight into how to choose the appropriate interface for a given class of applications. For example, for some applications, the limitation described in Section 3.4 may not be important, while for others, the limitation may cause design outside of potential games to be preferable.

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