Energy Procurement Strategies in the Presence of Intermittent Sources

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ABSTRACT

The increasing penetration of intermittent, unpredictable renewable energy sources such as wind energy, poses significant challenges for utility companies trying to incorporate renewable energy in their portfolio. In this work, we study the problem of conventional energy procurement in the presence of intermittent renewable resources. We model the problem as a variant of the newsvendor problem, in which the presence of renewable resources induces supply side uncertainty, and in which conventional energy may be procured in three stages to balance supply and demand. We compute closed form expressions for the optimal energy procurement strategy and study the impact of increasing renewable penetration, and of proposed changes to the structure of electricity markets. We explicitly characterize the impact of a growing renewable penetration on the procurement policy by considering a scaling regime that models the aggregation of unpredictable renewable sources. A key insight from our results is that there is a separation between the impact of the stochastic nature of this aggregation, and the impact of market structure and forecast accuracy. Additionally, we study the impact on procurement of two proposed changes to the market structure: the addition and the placement of an intermediate market. We show that addition of an intermediate market does not necessarily increase the efficiency of utilization of renewable sources. Further, we show that the optimal placement of the intermediate market is insensitive to the level of renewable penetration.

1. INTRODUCTION

Society's insatiable appetite for energy and growing environmental concerns have led many states in the United States to enact renewable portfolio standards. The standards mandate that utility companies must procure a certain percentage of their electricity from renewable sources [39]. For example, California has set the goal that 33% of its electricity should come from renewable sources by 2020. Among possible renewable sources, wind energy is expected to play a major role. There has been an explosive growth in installed wind capacity over the last few years [15] due to the ease of installation and low operational costs.

However, current electricity markets that govern energy procurement were designed for a scenario where there is very little uncertainty. More specifically, until now, supply side uncertainty has been low, arising mainly due to generator failures, which are rare. Furthermore, accurate demand forecasting ensures that the uncertainty in demand is small. However, going forward, the introduction of large volumes of highly intermittent and unpredictable renewable generation will increase supply side uncertainty dramatically. Thus, incorporating wind energy[1] into the energy portfolio of utilities is a challenging task that requires rethinking how electricity is procured [34, 12, 9]. *This paper seeks to provide insights into the impact of increasing supply side uncertainty on the 'efficiency' of procurement.*

Utility companies typically procure electricity via two modes of operation - *bilateral long term contracts* and *competitive electricity markets* [22, 35, 16]. In the former, utility companies sign long term bilateral contracts with various generators to supply certain amounts of electricity for specified periods. Currently, most utility companies purchase the bulk of their generation through long term bilateral contracts. This is feasible because the aggregate demand is highly predictable and because most conventional generators have very little uncertainty. To account for daily (or hourly) fluctuations in demand, the utility companies purchase the remainder of their electricity in competitive electricity markets. These markets, in which utility companies are buyers, and generators are sellers, are run by a third party called the independent service operator (ISO). There are typically two markets: a day-ahead or forward market, and a real time or spot market. A utility company may buy electricity in both markets in order to ensure that it has enough supply to meet the demand.

Integration of wind energy into current electricity markets has attracted considerable attention in recent years; excellent surveys can be found in [10, 34]. Broadly speaking, there are two different approaches to integrating wind into current electricity markets. In one approach, wind power producers participate only in competitive electricity markets e.g., California's participating intermittent resource program (PIRP). Such a scenario has been analyzed, for instance, in [5]. In the second approach, the wind power producers do not participate in electricity markets; instead, they sign long term multi-year contracts with utility companies. In such contracts, the utility company acquires rights to the energy generated from a certain wind farm installation in return for a predetermined payment. For example, Southern California Edison (SCE), a major utility company serving the greater Los Angeles area, has signed various contracts spanning $2-10$ years with various wind farms to procure wind energy ranging from 66.6 MW to 115 MW [30].

In this paper, we study the consequences of following the second approach. In particular, we study a setting where utility companies have procured large volumes of intermittent, unpredictable renewable energy via long term contracts. Because the realized amount of wind energy is variable and unpredictable, the utility company must still procure conventional generation via the electricity markets. Our goal is to study the impact of this long term commitment to renewable generation on the procurement strategies for con-

[1]For the remainder of the paper, we will use wind energy interchangeably with renewable energy; the models and the insights of our paper apply to any form of intermittent resource.

ventional generation.

Given the complexity of electricity procurement, we must consider a simplified model to be able to obtain analytical results. To that end, we consider a setting that ignores many complexities of generation and transmission (e.g., ramp constraints and line capacities), but models explicitly the multi-timescale nature of electricity procurement. Moreover, we assume that the utility company is a price-taker in the markets for conventional generation, i.e., it cannot influence the prices in these markets through its actions. These assumptions are standard in the literature and, though arguable, they enable us to derive a closed form characterization of the optimal energy procurement strategy of the utility company, which leads to several useful, counter-intuitive insights.

Contributions of this paper. The main contributions of this work fall, briefly, into two categories: (i) we characterize the optimal procurement strategy in the presence of long-term contracts for intermittent, unpredictable generation; and (ii) we study the impact of increasing renewable penetration and proposed changes to market structure on the optimal procurement strategy. We describe these each in more detail in the following.

The **first contribution** of this work is to characterize the optimal procurement strategy for a utility company that has a long term contract with an unpredictable generation source, e.g., wind energy (see Section 3). More specifically, we derive closed-form formulas for the optimal procurement that a utility company needs to make in both long term and day ahead markets. This result is a generalization of solutions for the classical newsvendor problem [1, 33, 20]. A key feature of our result is that the optimal procurement quantities can be viewed in terms of *reserves*, where these reserve quantities are the additional purchases that the utility needs to make to balance the current uncertainty of the supply and the higher cost of procuring energy in future markets.

The **second contribution** of this work is to study how the optimal procurement strategy changes as the penetration of renewable energy grows (see Section 4). In particular, we consider a scenario where the quantity of renewable generation contracted for grows, and ask how the procurement changes. The scaling for increasing penetration that we consider allows for a wide variety of models for how the unpredictability of renewable generation changes with increased penetration. For example, it includes scaling via additional sources with either independent or highly correlated generation. Our main result from this section yields a simple, informative equation summarizing the impact of renewable generation on the procurement of conventional generation. Specifically, Theorem 2 states that the average total procurement of conventional generation in the presence of a long term contract for wind is

$$d - \alpha\gamma + \delta\gamma^{\theta}$$

where d is the demand, α is the average generation of a single wind farm, γ is the number of wind farms (i.e., the renewable penetration), θ is a constant capturing the dependence between the generations of different wind farms, and δ is a constant that depends on the details of the market structure.

The way to interpret this equation is as follows. $d - \alpha\gamma$ represents the minimum average procurement, since this is the amount of demand that is not met by the wind. Thus, the 'extra' generation required because of the uncertainty of the wind is $\delta\gamma^{\theta}$. The key point about this term is that γ^{θ} is purely dependent on the degree of renewable penetration and the correlation between renewable sources; thus the impact of market structure is limited to δ.

The **third contribution** of this work is to study the impact of proposed changes to market structure [34, 9] on the optimal procurement strategy. In particular, there are two types of changes to the market structure that are most commonly considered: changing the *placement* of the day ahead market, e.g., by moving it closer to real time; and *adding markets*, e.g., adding a new market between the day ahead and real time markets [28]. The results in Sections 5 and 6 address the impact each of these possibilities.

First, Section 5 studies the impact of the placement of markets on procurement. This is a particularly salient issue because one might expect that as the penetration of renewable energy increases, it is beneficial to shift markets closer to real time, in order to take advantage of the improved prediction accuracy of the renewable generation. Our results highlight that this intuition may not be true. Specifically, we prove that, under very general assumptions, the placement of the day ahead market that minimizes the average total cost of procurement is independent of the penetration of renewables (Theorem 3).

Next, Section 6 studies the impact of additional markets on procurement. The addition of markets is often suggested as a way to help incorporate renewable generation by providing new markets closer to real time where predictions about renewable availability are more accurate. Our results highlight that one needs to be careful when considering such a change. Specifically, we contrast procurement in a two level market with procurement in a three level market in order to understand the role of adding an intermediate market. Of course, the *cost* of procurement always decreases as additional markets are introduced. However, with environmental concerns in mind, the key question is not about cost but about the *amount* of conventional generation procured. Perhaps surprisingly, additional markets do not always reduce the amount of conventional generation procured. Specifically, if we consider the addition of an intermediate market, then the average amount of conventional generation may drop or grow depending on the quality of the estimates for renewable generation, i.e., δ in Equation (1) may decrease or increase. Informally, if the estimation error is, in a sense, light-tailed (e.g., Gaussian), then the addition of an intermediate market reduces procurement of conventional generation (Theorem 5); but if the error has a heavy-tail (e.g., power-law, heavy-tailed Weibull), then the addition of an intermediate market can have the opposite effect (Theorem 4). Interestingly, it is typical to assume in analytical work that forecast errors are Gaussian [18, 38, 28], whereas empirical work on wind power generation suggests that a Weibull distribution may be a more accurate description [19, 6].

2. MODEL

Our goal in this paper is to understand how the presence of long term contracts for intermittent, unpredictable renewable generation impacts the procurement of conventional generation. Such long term contracts are a common, effective way of incorporating renewable energy into a utility's portfolio [30]; however they create challenges for a utility company's procurement of conventional generation. Thus, at the core of the paper, is a model of the electricity markets for conventional generation utility companies participate in, which we describe in this section. The key features we seek to capture are (i) the multi-timescale nature of electricity markets, (ii) price volatility, and (iii) the uncertainty of renewable generation.

Specifically, the procurement of conventional generation typically happens through participation in a multi-tiered set of electricity markets including a 'long-term market' (typically bilateral contracts), which could take place years or months ahead of time; an 'intermediate market', a.k.a. forward market, which could take place a day or several hours ahead of time; and a 'real time market', a.k.a. spot market. This multi-tiered structure means that when the utility company purchases conventional generation in the long term or intermediate markets it does not know how much renewable generation will be realized, nor does it know what the price will be in the spot market. So, its decisions must be made using only forecasts of these quantities, and, given the volatility of both renewable generation and prices, this creates a challenging procurement problem for the utility.

2.1 Model overview

To keep the model simple enough to allow analytic study, we ignore issues such as generator ramping constraints and transmission network capacity constraints in our model and further assume that the utility company has no access to large scale energy storage capacity. These assumptions allow us to focus on a single instant of time, which we denote by $t = 0$, and to consider only aggregate supply and demand.

We denote the electricity demand the utility company under consideration faces by d, and assume it is fixed and known ahead of time. This assumption is not restrictive in our setting, since demand uncertainty can be incorporated into the uncertainty of renewable generation. To meet the demand d, the utility company combines long term contracts for renewable generation (for simplicity we will often refer to this simply as "wind") with participation in a typical three-tier set of electricity markets for conventional generation:

(i) A *long term market*, in which the purchase commitment is made at time $-T_{lt}$. The price in this market is denoted p_{lt}, and when making its purchase commitment, the utility has a forecast $\hat{w}_{lt}$ of the wind generation that will be realized at $t = 0$.

(ii) An *intermediate market* (forward market), in which the purchase commitment is made at time $-T_{in}$. The price in this market is denoted p_{in}, and when making its purchase commitment, the utility has a forecast $\hat{w}_{in}$ of the wind generation that will be realized at $t = 0$.

(iii) A *real time market* (spot market), in which the purchase commitment is made at time $t = 0$. The price in this market is denoted p_{rt}, and utility knows the actual realization of the wind generation w at this time.

Of course, $-T_{lt} < -T_{in} < 0$.

The key feature of this model is the evolution of prices (p_{lt}, p_{in}, p_{rt}) and renewable forecasts $(\hat{w}_{lt}, \hat{w}_{in}, w)$ across markets. We describe our stochastic models for these evolutions in the following.

2.2 Evolution of prices

Prices in electricity markets are typically uncertain and volatile. Thus, for example, when deciding the procurement strategy in the long term market, a utility company does not know what the prices will be in the intermediate or real time markets. However, in general, conventional energy tends to be more expensive in markets closer to real time. The reason for this is that the marginal costs of production tend to be higher in spot markets than in forward or long term markets because any conventional energy that is demanded closer to real time is provided by generators that have low start up time and these generators typically are more expensive than generators that require several hours to start up.

Our model for the evolution of prices across the markets focuses on the two features described above – price volatility and increasing costs closer to real time.

Specifically, we assume that p_{lt} is the known *fixed* price in the long term markets. Since long term purchase commitments are made via bilateral contract where the utility company knows the price, this assumption is very mild and typically true in reality.

Next, let p_{in} be the *random* price in the intermediate market. We make the assumption that $\mathbb{E}[p_{in}] > p_{lt}$. This assumption reflects the fact that the generators used to supply electricity in the intermediate markets tend to have a higher marginal cost of production, and hence on average, the price in the intermediate market is higher that the price in long term market [31]. Note that the utility company knows the *exact* realization of the intermediate price at time of procurement in the intermediate market (i.e., at time $-T_{in}$). However, when the utility company is making a purchase commitment in the long term market, it is uncertain about the price in the intermediate market.

Similarly, let p_{rt} be the *random* price in the real time (or spot) market. We make the assumption that $\mathbb{E}[p_{rt}|p_{in}] > p_{in}$. This assumption states that given any realization of the intermediate price, the real time prices are higher on average. This reflects the fact that the electricity generated in real time market comes from fast ramp up generators which have a higher marginal cost of production. Note that at $t = 0$, the utility company knows the exact realization of the real time price, but its value is uncertain at the time of the long term and intermediate purchase commitments.

Importantly, the assumptions on the price evolution imply that $\mathbb{E}[p_{rt}] > \mathbb{E}[p_{in}] > p_{lt}$. However, any particular realization of the prices may have the price in real time market less than the intermediate price, or the price in the intermediate market less that the long term price.

Additionally, we make the following mild regularity assumptions. Let $[\underline{p}_{in}, \bar{p}_{in}]$ denote the support of the random variable p_{in}, where $\underline{p}_{in} > 0$. We assume that p_{in} is associated with a density function $f_{p_{in}}(\cdot)$ which is continuous over $(\underline{p}_{in}, \bar{p}_{in})$, and $f_{p_{in}}(p) > 0$ for $p \in (\underline{p}_{in}, \bar{p}_{in})$. Also, $\mathbb{E}[p_{rt}|p_{in} = p]$ is continuous with respect to p over $(\underline{p}_{in}, \bar{p}_{in})$. Finally, we assume that $\mathbb{E}[p_{in}], \mathbb{E}[p_{rt}] < \infty$.

Our model is a generalization of the well-known martingale model of forecast evolution [17, 14, 18], which assumes additionally that the random variables $p_{lt} - p_{in}$ and $p_{in} - p_{rt}$ are independent and normally distributed.

Finally, it is important to note that our model for price evolution assumes that the utility company cannot impact the price. This corresponds to assuming that the utility company under consideration is a small participant in the market, and hence is a 'price-taker'. This is a common assumption in literature [5, 21] and is typically true if there is enough competition from the demand side in electricity markets. Of course it would also be interesting to consider a more general model where the prices arrive endogenously from market behavior. However, incorporating multiple forward markets into multi-stage models is notoriously difficult and, as a result, the assumption of perfect foresight (no prediction error) is typically needed to obtain analytic results in these cases, e.g., [26, 7].

2.3 Evolution of wind forecasts

A fundamental challenge of incorporating long-term contracts for renewable energy is the uncertainty about how much renewable energy will be realized in real time and the fact that the utility company must guarantee that it procures enough generation to meet the demand despite this uncertainty. In particular, at the time of each market, the utility company needs to forecast how much wind generation will be available at time $t = 0$. Note that if the available wind energy was known with certainty, the utility company could purchase its entire remaining demand exactly in the long term market; however, because the amount of available wind energy at time $t = 0$ is highly uncertain at the time of the long term market, the utility company needs to balance its lowest cost purchase in long term with better estimates of wind energy closer to real time.

To capture this, we consider a model where the forecast accuracy of the wind generation improves upon moving closer to real time, i.e., $t = 0$.

$$h(r) \triangleq p_{lt} - \int_{p=\underline{p}_{in}}^{\bar{p}_{in}} p\bar{F}_{\mathcal{E}_1}(r - r_{in}(p)) f_{p_{in}}(p)\ dp - \int_{p=\underline{p}_{in}}^{\bar{p}_{in}} \mathbb{E}[p_{rt}|p_{in}=p] P\left(\mathcal{E}_1 + \mathcal{E}_2 > r;\ \mathcal{E}_1 < r - r_{in}(p)\right) f_{p_{in}}(p)\ dp = 0 \quad (6)$$

Specifically, we assume that the utility company comes to know the value of wind realized at time $t = 0$, denoted by w, only when purchasing generation in the real time market. During the long term and intermediate markets the utility only has estimates of w, denoted by $\hat{w}_{lt}$ and $\hat{w}_{in}$ respectively. Clearly, in general, uncertainty about the wind generation w decreases as one moves closer to real time. Formally, we capture this by assuming that

$$\hat{w}_{in} = \hat{w}_{lt} - \mathcal{E}_1, \text{ and } w = \hat{w}_{in} - \mathcal{E}_2, \qquad (1)$$

where $\mathcal{E}_1$ and $\mathcal{E}_2$ are zero mean independent random variables, independent of the prices p_{in} and p_{rt}. In other words, we assume that the forecast of w evolves with *independent increments* where the random variables $\mathcal{E}_1$ and $\mathcal{E}_2$ capture these increments. Note that $\hat{w}_{lt}$ is a coarser estimate of w than $\hat{w}_{in}$, since the long term forecast error $\hat{w}_{lt} - w = \mathcal{E}_1 + \mathcal{E}_2$ is more variable than the intermediate forecast error $\hat{w}_{in} - w = \mathcal{E}_2$. Our model is a generalization of the well-known martingale model of forecast evolution [17, 14, 18], which makes the additional assumption that $\mathcal{E}_1$ and $\mathcal{E}_2$ follow a Gaussian distribution.

Let $[L_1, R_1]$ and $[L_2, R_2]$ denote respectively the supports of the random variables $\mathcal{E}_1$ and $\mathcal{E}_2$, where $L_1, L_2 \in \{-\infty\} \cup \mathbb{R}$ and $R_1, R_2 \in \mathbb{R} \cup \{\infty\}$. We make the following regularity assumptions on the distributions of $\mathcal{E}_1$ and $\mathcal{E}_2$: they are associated with continuously differentiable density functions, denoted by $f_{\mathcal{E}_1}(\cdot)$ and $f_{\mathcal{E}_2}(\cdot)$ respectively, with $f_{\mathcal{E}_i}(x) > 0$ for $x \in (L_i, R_i)$.

This concludes our discussion of the model of this paper. The following notation is used heavily in the remainder of this paper. We use $P(E)$ to denote the probability of an event E, and $\bar{F}_X$ to denote the complementary cumulative distribution function associated with the random variable X, i.e., $\bar{F}_X(x) = P(X > x)$. Finally, $[x]_+ := \max\{x, 0\}$.

3. OPTIMAL PROCUREMENT

In this section, we formalize the utility's procurement problem, and then characterize the optimal procurement strategy. This strategy is the basis for the explorations of the impact of increased penetration of renewables and changes to market structure in subsequent sections.

3.1 The procurement problem

To begin, note that the procurement decision of the utility in each market can depend only on the information available to the utility company at that time. Specifically, in each market, we consider procurement strategies that depend only on (i) the wind estimate available at the time of purchase, (ii) the price of conventional generation in the current market, and (iii) the total conventional generation that has already been procured.[2] Accordingly, let $q_{lt}(\hat{w}_{lt}, p_{lt})$ denote the quantity of conventional generation procured in the long term market, given the long term wind estimate $\hat{w}_{lt}$ and long term price p_{lt}. Similarly, let $q_{in}(\hat{w}_{in}, q_{lt}, p_{in})$ denote the quantity of conventional generation procured in the intermediate market, given the corresponding wind estimate $\hat{w}_{in}$, the realized price p_{in}, and q_{lt} (the quantity procured already). Finally, let $q_{rt}(w, q_{lt} + q_{in}, p_{rt})$ denote the the quantity of conventional generation procured in the real time market, which depends on the realized wind w, the actual price in the real time market p_{rt}, and the quantity procured already, i.e., $q_{lt} + q_{in}$. For notational convenience, we often drop the arguments from these functions and simply write q_{lt}, q_{in}, q_{rt}. We make the assumption that the utility cannot sell power in any market[3], and thus the quantities procured in all three markets must be non negative.

When determining these procurement quantities, we assume that the utility company is seeking to minimize its expected total cost while ensuring that the total quantity purchased in these three markets satisfies the residual demand (i.e., the demand minus the available wind in real time). Thus, we can express the optimal procurement problem as follows:

$$\min_{q_{lt}, q_{in}, q_{rt}} \ \mathbb{E}\left[p_{lt}q_{lt} + p_{in}q_{in} + p_{rt}q_{rt}\right] \qquad (P)$$

$$\text{subject to } q_{lt} \geq 0,\ q_{in} \geq 0,\ q_{rt} \geq 0$$
$$q_{lt} + q_{in} + q_{rt} + w \geq d.$$

Recall that q_{lt}, q_{in} and q_{rt} are functions that depend on the corresponding wind estimate, price of conventional generation, and the total procurement so far. Here, the expectation is with respect to the randomness associated with the wind forecast evolution and price volatility.

The optimal procurement problem posed above is mathematically equivalent to a variant of the classical newsvendor problem [20, 27]. In Section 7, we discuss the relationship between our work and the literature on the newsvendor problem.

A final comment about the the optimal procurement problem above is that we have considered a three tiered market structure that models the common current practice. In general, the optimal procurement problem is simply a Markov decision process, and can be studied for arbitrary numbers of tiers, e.g., see [28]. We limit ourselves to a three tiered structure in this paper to keep the analysis simple and the resulting formulas interpretable.

3.2 The optimal procurement strategy

The following theorem is the foundational result for the remainder of the paper. It characterizes the optimal procurement strategy for a utility company in a three tiered market with price volatility.

THEOREM 1. *The optimal procurement strategy for the utility company in the three tiered market scenario of Problem* (P) *is:*

$$q_{lt}^* = [d - \hat{w}_{lt} + r_{lt}]_+ \qquad (2)$$
$$q_{in}^* = [d - \hat{w}_{in} - q_{lt} + r_{in}(p_{in})]_+ \qquad (3)$$
$$q_{rt}^* = [d - w - q_{lt} - q_{in}]_+\,, \qquad (4)$$

where

$$r_{in}(p_{in}) = \bar{F}_{\mathcal{E}_2}^{-1}\left(\frac{p_{in}}{\mathbb{E}[p_{rt}|p_{in}]}\right), \qquad (5)$$

and r_{lt} *is the unique solution of* (6).

The proof of Theorem 1 is given at the end of this section. A key feature of this theorem is that the structure of

[2]Given our stochastic model for prices and wind forecast evolution, it is easy to show that there is no loss of optimality in restricting policies to this class.

[3]Our model can be extended to relax this assumption. However, we do not consider this generalization because it adds complexity without providing additional insight.

the the optimal procurement strategy gives a natural interpretation to r_{lt} and $r_{in}(p_{in})$ as *reserve levels.* Specifically, at the time of purchase in the long term market, $d - \hat{w}_{lt}$ can be interpreted as an estimate of the conventional procurement that is required to meet the demand. Then r_{lt} is the additional 'reserve' purchased by the utility to balance the current wind uncertainty and the higher cost of conventional energy in subsequent markets. The reserve $r_{in}(p_{in})$ has a similar interpretation. We note that the intermediate reserve level is a function of the price of conventional generation in the intermediate market. For notational simplicity, we often drop the argument of this function, and simply write r_{in}. We henceforth refer to r_{lt} and r_{in} as the (optimal) long term and intermediate reserves respectively.

It is important to point out that the reserves r_{lt} and r_{in} may be positive or negative. A negative reserve implies that it is optimal for the utility to maintain a net procurement level that is *less* than the currently anticipated residual demand, and purchase any shortfall in subsequent markets. The values of the optimal reserves depend on the price volatility (via the distribution of intermediate and real time prices) as well as the accuracy of the wind forecasts $\hat{w}_{lt}$ and $\hat{w}_{in}$ (via the distributions of the random variables $\mathcal{E}_1$ and $\mathcal{E}_2$). Note that the optimal reserves do not depend on the demand d, or on procurements made in prior markets.

As is evident from Equation (5), the optimal reserve in the intermediate market is structurally similar to the critical fractile solution of the classical newsvendor model [1]. This is because, at the time of the intermediate market, the utility company conjectures the average real time price to be $\mathbb{E}[p_{rt}|p_{in}]$, and having already purchased the quantity q_{lt}, it faces a problem similar to the classical newsvendor problem.

Finally, Theorem 1 highlights the need for the following additional assumption in order to avoid trivial solutions.

ASSUMPTION 1. *We assume that the demand is large enough that the utility company procures a positive quantity in the long term market. That is,* $d - \hat{w}_{lt} + r_{lt} > 0$.

This assumption ensures that the optimal procurement problem in the three market has a non-trivial solution in the following sense: if $d - \hat{w}_{lt} - r_{lt} \leq 0$, then the utility company procures $q^*_{lt} = 0$ in the long term. In this case, the procurement problem effectively reduces to a two market scenario. Intuitively, Assumption 1 will hold as long as the demand d exceeds the peak capacity of the renewable installations, which is of course true in most current practical scenarios. Assumption 1 allows us to rewrite the optimal procurement quantities in three markets as follows:

$$\begin{aligned} q^*_{lt} &= d - \hat{w}_{lt} + r_{lt}, \\ q^*_{in} &= [\mathcal{E}_1 - r_{lt} + r_{in}(p_{in})]_+ , \\ q^*_{rt} &= [\mathcal{E}_2 - r_{lt} + \min\{\mathcal{E}_1, r_{lt} - r_{in}(p_{in})\}]_+ , \end{aligned} \tag{7}$$

where we abuse notation to let q^*_{lt}, q^*_{in} and q^*_{rt} represent the (random) quantities that are purchased in the long term, the intermediate, and the real time markets respectively.

We conclude this section with the proof of Theorem 1.

PROOF OF THEOREM 1. Since the utility is required to procure enough generation to satisfy its demand, it it easy to see that the optimal strategy in real time is to procure just enough conventional energy to *exactly* meet the demand, i.e., $q^*_{rt} = [d - w - q_{lt} - q_{in}]_+$. This ensures that the last constraint in the optimization (P) is always satisfied.

Having decided the optimal strategy for real time, the optimization problem (P) can now be re-written as

$$\min_{q_{lt}, q_{in}} \mathbb{E}\left[p_{lt}q_{lt} + p_{in}q_{in} + p_{rt}\left[d - w - q_{lt} - q_{in}\right]_+\right]$$
$$\text{subject to } q_{lt} \geq 0,\ q_{in} \geq 0.$$

This problem is a 3-stage Markov decision process [4], with the stages corresponding to the long term, intermediate, and real time markets. The state in each market is the tuple consisting of (i) the current wind estimate, (ii) the price of conventional generation in the current market, and (iii) the total conventional generation procured so far. The action in each stage corresponds to the procurement decision in that market, and the stage cost is the cost of that procurement. We summarize the Markov decision process associated with the optimal procurement problem in the table below.

stage	state	action	stage cost
1	$S_1 = (\hat{w}_{lt}, p_{lt}, 0)$	q_{lt}	$p_{lt}q_{lt}$
2	$S_2 = (\hat{w}_{in}, p_{in}, q_{lt})$	q_{in}	$p_{in}q_{in}$
3	$S_3 = (w, p_{rt}, q_{lt} + q_{in})$		$p_{rt}\left[d - w - q_{lt} - q_{in}\right]_+$

We can now compute the optimal procurement strategy for the intermediate and the long term markets using the dynamic programing algorithm [4]. In the intermediate market, the optimal procurement is the minimizer of the expected *cost to go*; it is therefore the solution of the following optimization.

$$\min_{q_{in} \geq 0} \quad \mathbb{E}\left[p_{in}q_{in} + p_{rt}\left[d - w - q_{lt} - q_{in}\right]_+ \mid S_2\right]. \tag{8}$$

Using Equation (1) to write $w = \hat{w}_{in} - \mathcal{E}_2$ and making the substitution $q_{in} = d - q_{lt} - \hat{w}_{in} + r$, we can write the objective function in the above minimization problem as

$$\xi(r) = p_{in}(d - q_{lt} - \hat{w}_{in} + r) + \mathbb{E}[p_{rt}|p_{in}]\ \mathbb{E}\left[\mathcal{E}_2 - r\right]_+ .$$

Here, we can think of r as the additional reserve required in the intermediate market. By direct differentiation, we get that $\xi'(r) = p_{in} - \mathbb{E}[p_{rt}|p_{in}]\ \bar{F}_{\mathcal{E}_2}(r)$. Since $\xi'(r)$ is non-decreasing, $\xi(\cdot)$ is convex. Moreover, $r_{in}(p_{in}) \in (L_2, R_2)$ is the unique minimizer of $\xi(\cdot)$ over $\mathbb{R}$. It is now easy to see that minimization (8) is convex, and that the optimal procurement in the intermediate market is given by (3).

The derivation of the optimal procurement in the long term market is more cumbersome. Due to space constraints, we omit some algebraically intensive calculations here, and only outline the main steps of the derivation. Denoting the optimal *cost to go* from the intermediate stage onwards by $J^*_2(S_2)$, the optimal procurement in the long term market is the solution to the following optimization.

$$\min_{q_{lt} \geq 0} \quad \mathbb{E}\left[p_{lt}q_{lt} + J^*_2(S_2) \mid S_1\right] \tag{9}$$

In the above objective, we make the substitution $q_{lt} = d - \hat{w}_{lt} + r$, with the interpretation that r is the additional reserve in the long term. Denoting the objective now by $\varphi(r)$, it can be shown that

$$\varphi'(r) = h(r),$$

where $h(r)$ is defined in Equation (6). Differentiating again, we obtain

$$\begin{aligned} \varphi''(r) &= h'(r) \\ &= \int_p \int_{r_{in}}^{\infty} \mathbb{E}[p_{rt}|p_{in} = p]\ f_{\mathcal{E}_1}(r - z)\ f_{\mathcal{E}_2}(z)\ f_{p_{in}}(p)\ dz\ dp. \end{aligned}$$

Here, the integral with respect to the variable p is over the interval $[\underline{p}_{in}, \bar{p}_{in}]$, i.e., the support of the random intermediate price p_{in}. Also, we interpret a density function to be zero outside the support of the corresponding random variable, if the support is finite. Since $\varphi''(\cdot)$ is non-negative, $\varphi(\cdot)$ is convex. Let $\underline{r}_{in} := \inf_{p \in (\underline{p}_{in}, \bar{p}_{in})} r_{in}(p)$. Since $f_{\mathcal{E}_i}(x) > 0$ over $x \in (L_i, R_i)$, it follows that $\varphi''(r) > 0$ for $r \in (L_1 + \underline{r}_{in}, R_1 + R_2)$, and that $\varphi''(r) = 0$ for all $r < L_1 + \underline{r}_{in}$ and $r > R_1 + R_2$. This implies that $\varphi'(r)$ is *strictly* increasing over the range $r \in (L_1 + r_{in}, R_1 + R_2)$. An elementary application of the dominated convergence theorem yields

$$\lim_{r\to-\infty} \varphi'(r) = p_{lt} - \mathbb{E}[p_{in}] < 0, \qquad \lim_{r\to\infty} \varphi'(r) = p_{lt} > 0.$$

It therefore follows that the equation $\varphi'(r) = 0$ has a unique solution $r_{lt} \in (L_1 + \underline{r}_{in}, R_1 + R_2)$. Moreover, r_{lt} is the unique minimizer of $\varphi(\cdot)$ over $\mathbb{R}$. Finally, it follows as before that the optimization (9) is convex, and that the optimal procurement in the long term market is given by (2). $\square$

4. THE IMPACT OF INCREASING RENEWABLE PENETRATION

The penetration of renewable energy, in particular of wind energy, is poised for major growth over the coming years. A consequence of this is that utility companies will face an ever increasing supply side uncertainty. In this section, we explore the impact of this growth on procurement. Specifically, we ask: *how will the optimal procurement policy change as the volume of intermittent wind resources increases?*

To answer this question, we introduce a scaling regime for wind penetration, which models the effect of aggregating the output of several wind generators. A key feature of our scaling model is that it allows for varying levels of stochastic dependence between the intermittent energy sources being aggregated. For example, our model lets us study the aggregation of independent sources, as well as perfectly correlated sources.

Based on this scaling model, we study how the optimal reserves, the amount of conventional generation procured, as well as the cost of procurement scale with increasing wind penetration. Our analysis yields clean and easy to interpret scaling laws for these quantities. Remarkably, the scaling laws reveal a decoupling between the impact of the level of stochastic dependence between different wind sources and the impact of market structure and wind forecast accuracy.

4.1 A scaling regime

We begin by describing our scaling model for wind penetration. We start with a baseline scenario. Let us denote by α the average output of a single intermittent generator. For concreteness, we refer to this as the baseline wind farm in the following. We let $\tilde{\mathcal{E}}_1$ and $\tilde{\mathcal{E}}_2$ be the error random variables that relate the long term wind estimate $\hat{w}_{lt}$ to the estimate of the wind in the intermediate market $\hat{w}_{in}$ and the actual wind realization w (see Equation (1)). We assume that in the long term market there is no better estimate of the wind than the long term average, i.e., $\hat{w}_{lt} = \alpha$. Note that the optimal procurement in the baseline scenario (before scaling) can be computed using Theorem 1. For the remainder of this section, we use $\tilde{r}_{lt}$ and $\tilde{r}_{in}$ to denote respectively the optimal long term and the intermediate reserve in the baseline scenario. We emphasize that $\tilde{r}_{in}$ is a function of the realization of the intermediate price p_{in}, although we suppress this dependence for notational convenience in this section.

To scale the wind penetration, we introduce a scale parameter γ that is proportional to the capacity of wind generation. The scale parameter γ may be interpreted as the number of (homogeneous) wind farms whose aggregate output is available to the utility company. Thus, the average wind energy available to the utility company is given by $\hat{w}_{lt}(\gamma) = \gamma\alpha$. Clearly, as the capacity of wind generation scales, the error in the wind forecast available at both the long term and the intermediate markets will also scale, and this scaling will depend on the correlation between the generation at each of the wind farms. We use the following simple, but general scaling model.

$$\begin{aligned} \hat{w}_{lt}(\gamma) &= \gamma\alpha, \\ \mathcal{E}_1(\gamma) &= \gamma^\theta \tilde{\mathcal{E}}_1, \\ \mathcal{E}_2(\gamma) &= \gamma^\theta \tilde{\mathcal{E}}_2, \end{aligned} \tag{10}$$

where we let $\theta \in [1/2, 1]$. It is important to point out that our scaling regime leaves the prices of conventional generation unchanged; we scale only the volume of intermittent generation.

To interpret this scaling, consider first the case $\theta = 1$. In this case, $\mathcal{E}_1 = \gamma\tilde{\mathcal{E}}_1$ and $\mathcal{E}_2 = \gamma\tilde{\mathcal{E}}_2$, implying that $\hat{w}_{in}(\gamma)$ and $w(\gamma)$ scale proportionately with γ. This scaling corresponds to a scenario where the aggregate wind output with γ wind farms is simply γ times the output of the baseline wind farm. One would expect such a scaling to occur if the wind farms are co-located.

The case of $\theta = 1/2$ (with $\tilde{\mathcal{E}}_1$ and $\tilde{\mathcal{E}}_2$ being normally distributed) corresponds to a central limit theorem scaling, and seeks to capture the scenario where the output of each wind farm is independent. Equation (10) captures this scenario exactly if the forecast evolution distributions for each wind farm follow a Gaussian distribution. If not, Equation (10) can be interpreted as an approximation for large enough γ based on the central limit theorem. Intuitively, one would expect such a scaling if the different wind farms are geographically far apart.

Finally, the case $\theta \in (\frac{1}{2}, 1)$ seeks to capture correlations that are intermediate between independence and perfect correlation (see, for instance [2, 36]).

Note that the forecast error distributions grow slowest when the outputs of different wind farms are independent, and fastest when the outputs are perfectly correlated.

4.2 Scaling results

Given the scaling regime described above, we can characterize the impact of increasing penetration of intermittent resources on the procurement of conventional generation. First, we analyze how the optimal reserve levels scale with increasing wind penetration. Next, we use these results to obtain scaling laws for the procurement quantities in the three markets, and also the total procurement and the total cost of procurement.

The scaling of reserve levels.

The following lemma characterizes the scaling of the optimal reserves under our wind scaling model. It shows the optimal reserve levels in the long term and intermediate markets follow the same scaling as imposed on the distributions of the forecast errors.

Lemma 1. *Under the scaling regime defined in Equation* (10)*, the optimal long term and intermediate reserves scale as:*

$$r_{lt}(\gamma) = \gamma^\theta \tilde{r}_{lt}, \quad r_{in}(\gamma) = \gamma^\theta \tilde{r}_{in}. \tag{11}$$

We emphasize that $\tilde{r}_{in}$ and $r_{in}(\gamma)$ are both functions of the intermediate price. Equation (11) states that the function $r_{in}(\gamma)$ scales proportionately to γ^θ, i.e., $(r_{in}(\gamma))(p_{in}) = \gamma^\theta \tilde{r}_{in}(p_{in})$.

Proof. From Theorem 1, given any scale parameter γ, the optimal reserve in the intermediate market $r_{in}(\gamma)$ is the unique solution of the equation

$$P\left(\mathcal{E}_2(\gamma) > r_{in}(\gamma)\right) = \frac{p_{in}}{\mathbb{E}[p_{rt}|p_{in}]}. \tag{12}$$

Now, noting that $\mathcal{E}_2(\gamma) = \gamma^\theta \tilde{\mathcal{E}}_2$, and $P\left(\tilde{\mathcal{E}}_2 > \tilde{r}_{in}\right) = \frac{p_{in}}{\mathbb{E}[p_{rt}|p_{in}]}$ (by Theorem 1 applied to the baseline scenario), we conclude that $r_{in}(\gamma) = \gamma^\theta \tilde{r}_{in}$ satisfies Equation (12).

The optimal long term reserve $r_{lt}(\gamma)$ is the unique solution of Equation (6), with $\mathcal{E}_1$, $\mathcal{E}_2$, and r_{in} substituted by $\mathcal{E}_1(\gamma)$, $\mathcal{E}_2(\gamma)$, and $r_{in}(\gamma)$, respectively. As before, using the characterization of $\tilde{r}_{lt}$ obtained from Equation (6) applied to the baseline scenario, it is easy to verify that $r_{lt}(\gamma) = \gamma^{\theta}\tilde{r}_{lt}$. $\square$

The scaling of procurement quantities.

Using Lemma 1, we can now characterize the optimal procurement amounts in the three markets. As in Section 3, to avoid the trivial solution with zero long term procurement, we restrict the range of scale parameter γ to satisfy

$$d > \gamma\alpha - \gamma^{\theta}\tilde{r}_{lt}, \tag{13}$$

This ensures that the optimal long term procurement, given by $q^*_{lt}(\gamma) = [d - \hat{w}_{lt}(\gamma) + r_{lt}(\gamma)]_+$ is strictly positive.[4] For the remainder of this section, we focus on the range of scale parameter γ that satisfies Condition (13). Intuitively speaking, Condition (13) will hold as long as the demand is larger than the peak wind capacity. Under this assumption, using Equation (7) and Lemma 1, we get that the optimal procurement quantities in the long term, intermediate, and real time markets is given by

$$\begin{aligned} q^*_{lt}(\gamma) &= d - \gamma\alpha + \gamma^{\theta}\tilde{r}_{lt}, \\ q^*_{in}(\gamma) &= \gamma^{\theta}\left[\tilde{\mathcal{E}}_1 - \tilde{r}_{lt} + \tilde{r}_{in}\right]_+, \\ q^*_{rt}(\gamma) &= \gamma^{\theta}\left[\tilde{\mathcal{E}}_2 - \tilde{r}_{lt} + \min\{\tilde{\mathcal{E}}_1, \tilde{r}_{lt} - \tilde{r}_{in}\}\right]_+. \end{aligned} \tag{14}$$

The above equations reveal that as we increase the penetration of wind, the optimal procurement in the intermediate and the real time markets increases proportionately to γ^{θ}. This increasing procurement in markets closer to real time is a consequence of the increasing uncertainty in the renewable forecasts. Indeed, note that the procurements in the intermediate and real time markets scale in exactly the same manner as the forecast error distributions. Therefore, these procurements scale slowest when outputs of different wind farms are independent, and fastest when the outputs are perfectly correlated. From the standpoint of the system operator, Equation (14) describes how the installed capacity of fast ramp conventional generators that can supply energy in the intermediate and real time markets needs to scale as the capacity of wind generation scales.

The scaling of the total procurement and the total cost of procurement.

As we scale the wind capacity, we expect that the amount of total conventional energy procured, as well as the cost of procurement must decrease. The following theorem characterizes the scaling laws for these quantities. Let $TP(\gamma)$ and $TC(\gamma)$ denote respectively the total procurement of conventional energy and the total cost of procurement, corresponding to scale parameter γ.

THEOREM 2. *For the range of scale parameter γ satisfying Equation (13), the expected total procurement and the expected total cost are given by*

$$\mathbb{E}[TP(\gamma)] = d - \alpha\gamma + \delta\gamma^{\theta}, \tag{15}$$

$$\mathbb{E}[TC(\gamma)] = p_{lt}(d - \alpha\gamma) + \delta'\gamma^{\theta}, \tag{16}$$

where, $\delta \geq 0$ and $\delta' \geq 0$ are defined as

$$\begin{aligned} \delta &\triangleq \tilde{r}_{lt} + \mathbb{E}\left[\tilde{\mathcal{E}}_1 - (\tilde{r}_{lt} - \tilde{r}_{in})\right]_+ \\ &\quad + \mathbb{E}\left[\tilde{\mathcal{E}}_2 - \tilde{r}_{lt} + \min\{\tilde{\mathcal{E}}_1, \tilde{r}_{lt} - \tilde{r}_{in}\}\right]_+, \\ \delta' &\triangleq p_{lt}\tilde{r}_{lt} + \mathbb{E}\left\{p_{in}\left[\tilde{\mathcal{E}}_1 - (\tilde{r}_{lt} - \tilde{r}_{in})\right]_+\right\} \\ &\quad + \mathbb{E}\left\{p_{rt}\left[\tilde{\mathcal{E}}_2 - \tilde{r}_{lt} + \min\{\tilde{\mathcal{E}}_1, \tilde{r}_{lt} - \tilde{r}_{in}\}\right]_+\right\}. \end{aligned}$$

The scaling law (15) has the following interpretation. If there were no uncertainty in the wind generation, i.e., $w(\gamma) = \hat{w}_{lt}(\gamma) = \alpha\gamma$, then it easy to see that the utility would procure the exact residual demand, i.e., $d - \alpha\gamma$ in the long term market (since the prices of conventional generation increase on average as we move closer to real time). From (15), we see that on average, the utility has to make an 'additional procurement' equal to $\delta\gamma^{\theta}$ as a result of the uncertainty in the wind generation. The fact that this additional procurement grows proportionately to γ^{θ} highlights the benefit of aggregating independent wind sources. Specifically, if $\theta = 1$, i.e., the wind farms are co-located, then the additional procurement is of the same order as the wind capacity. On the other hand, if $\theta = 1/2$, i.e., the wind farms are geographically far apart, then the additional procurement grows much slower than the wind capacity.

The scaling law (16) has a similar interpretation. As before, note that if there were no uncertainty in the wind generation, then $p_{lt}(d - \gamma\alpha)$ equals the (optimal) average cost of conventional procurement. From (16), we see that on average, the utility incurs an 'additional cost' equal to $\delta'\gamma^{\theta}$ as a result of the uncertainty in the wind generation. Once again, we see the benefit of aggregating independent wind sources. Thus, intuitively speaking, a utility company would benefit from signing long term contracts with wind generators that are as geographically spread out as possible.[5]

The structure of the scaling laws (15) and (16) also reveals an interesting separation between the impact of the level of stochastic dependence between different wind sources and the impact of market structure and wind forecast accuracy. To see this, consider the 'additional procurement' $\delta\gamma^{\theta}$ in Equation (15) and the 'additional cost' $\delta'\gamma^{\theta}$ in Equation (16). The factor γ^{θ} is purely dependent upon the volume of wind capacity and the stochastic dependence between the outputs of the different wind farms. On the other hand, the factors δ and δ' depend on only the prices of conventional energy in the three markets (and their volatility), and the baseline forecast error distributions. In other words, δ and δ' are invariant with respect to the aggregation of wind sources. As we discuss in the next section, this separation has interesting consequences for market design.

We now present the proof of Theorem 2, which follows from Lemma 1 and Theorem 1.

PROOF OF THEOREM 2. The expected total procurement and the expected total cost follow trivially from Equation (14).

To prove that $\delta \geq 0$, we note that the optimization problem P requires that the total procurement exceed $d - w(\gamma)$ for every realization of wind w. Thus, the expected total procurement must satisfy $\mathbb{E}[TP(\gamma)] \geq d - \gamma\alpha$, which implies, using Equation (15), that $\delta \geq 0$.

To prove that $\delta' \geq 0$, we consider a hypothetical procurement formulation with no wind uncertainty. Specifically, consider a scenario in which the values of $\mathcal{E}_1(\gamma)$ and $\mathcal{E}_2(\gamma)$ are revealed to the utility a priori. This implies that

[4] One sufficient θ-independent condition that ensures that Condition (13) holds is $\gamma < \frac{d}{\alpha + |\tilde{r}_{lt}|}$. This imposes an upper bound on the degree of penetration that we consider.

[5] This above discussion of course assumes implicitly that network capacity is not a bottleneck in the utilization of the available wind energy.

the utility knows the exact realization of the wind generation, and therefore the exact residual demand, before making its procurement decisions for conventional generation. This conventional generation can be procured in the same three-tiered market structure as in our original formulation, with the same price volatility.

The problem of optimal procurement in this hypothetical setting, seeking to minimize the average cost of procurement (and subject to satisfying the demand d), can be formulated as before. Since the prices are ordered on average, it is easy to show that the optimal policy in this case is to buy the entire residual demand (equal to $d - w(\gamma)$) in the long term market. However, yet another feasible strategy is to make procurements according to Equation (14). Clearly, the average cost for this latter strategy must be at-least the average cost of the optimal policy, i.e.,

$$\begin{aligned}&\mathbb{E}_{p_{in},p_{rt}}\Bigg(p_{lt}(d-\gamma\alpha+\gamma^{\theta}\tilde{r}_{lt})+p_{in}\gamma^{\theta}\left[\tilde{\mathcal{E}}_1-\tilde{r}_{lt}+\tilde{r}_{in}\right]_+\\&\qquad+p_{rt}\gamma^{\theta}\left[\tilde{\mathcal{E}}_2-\tilde{r}_{lt}+\min\{\tilde{\mathcal{E}}_1,\tilde{r}_{lt}-\tilde{r}_{in}\}\right]_+\Bigg)\\&\geq p_{lt}(d-w(\gamma)).\end{aligned}$$

Note that the expectation above is only taken with respect to the prices. Now, taking expectations with respect to $\mathcal{E}_1(\gamma)$ and $\mathcal{E}_2(\gamma)$, we conclude that $\mathbb{E}[TC(\gamma)] \geq p_{lt}(d - \alpha\gamma)$, which implies, using Equation (16), that $\delta' \geq 0$. This completes the proof. □

5. THE OPTIMAL PLACEMENT OF THE INTERMEDIATE MARKET

In current electricity markets, the intermediate market takes place about 14 hours before $t = 0$ (this market is called the day ahead or forward market) [8]. Because the demand can be predicted reasonably accurately by this time, this allows the utility to procure most of its generation much before the time of use. However, accurate prediction of wind in the current day ahead markets is not feasible. Thus, it is commonly suggested that as the penetration of wind increases, the system operator may decide to move the intermediate market closer to real time to allow for better prediction of the wind. In this section, we consider the optimal placement of the intermediate market and study how this optimal placement changes as we increase the penetration of wind energy.

Recall that, in our model, the intermediate market takes place T_{in} time units prior to the time of use (i.e. $t = 0$). If the system operator were to move this intermediate market closer to real time, this would imply better estimates of available wind at that time. However, in moving the market closer to real time, the procured conventional generation must come from generators that have faster ramp up times. These generators are typically more expensive and hence we would expect that the price of the conventional generation in the intermediate market would increase (on average) as we move the market closer to real time. We define the optimal placement of the intermediate market as the one that minimizes the average total procurement cost. Note that this optimal placement balances an improving forecast of the wind generation moving closer to real time with an increasing price of conventional energy.

The optimal placement is clearly a function of how the forecast errors $\tilde{\mathcal{E}}_1$ and $\tilde{\mathcal{E}}_2$ change as the function of the intermediate market time T_{in}. It also depends on how the joint distribution of the prices of conventional generation in the intermediate and the real time market changes as T_{in} changes. Let us denote by T_{in}^* the optimal placement of the intermediate market that minimizes the expected total cost of procuring conventional generation.

Importantly, Theorems 1 and 2 allow the computation of T_{in}^* numerically. However, there is little structural insight for this placement that can be provided. But, an important question that we can provide analytic insight about is the following: *how does the optimal placement of the intermediate market change as we increase the penetration of wind energy?*

The answer to this question has significant implications for future market design. Remarkably, based on the scaling regime developed in the previous section, the following theorem tells us that the optimal placement does not depend upon the scale parameter γ or the aggregation parameter θ.

THEOREM 3. *For the wind scaling satisfying Equation* (13)*, over the range of γ satisfying Condition* (13) *for all considered placements of the intermediate market, the optimal placement of the intermediate market is independent of the scale parameter γ and the correlation parameter θ.*

PROOF. From Theorem 2, we note that the expected total cost of procurement is given by

$$\mathbb{E}[TC(\gamma)] = p_{lt}(d - \alpha\gamma) + \delta'\gamma^{\theta}.$$

Furthermore, the effect of placement is only on the parameter δ' (via the distributions of $\tilde{\mathcal{E}}_1$, $\tilde{\mathcal{E}}_2$, p_{in}, and the prices p_{in} and p_{rt}) which is independent of γ and θ. Thus, the optimal placement is independent of scale parameter γ and the aggregation parameter θ. □

Theorem 3 has important implications for market design. It says that the optimal intermediate market placement can be decided independently of how many wind farms are there in the system, and how correlated their outputs are. Thus, the system operator can keep the placement of this market fixed as more wind energy is incorporated into the system.

6. THE VALUE OF ADDITIONAL FORWARD MARKETS

One of the key objectives of this paper is to study the impact of increased renewable penetration on the structure of electricity markets. As seen in the previous section, the optimal placement of the intermediate market is independent of the amount of wind present in the system. In this section, we look at another important market design question: *can we facilitate the penetration of renewable energy by providing additional forward markets?* It is commonly suggested that having additional markets would be beneficial, since this allows the utility to better exploit the evolution of the wind forecast. Indeed, the intuition is true in terms of procurement cost – having additional forward markets benefits the utility company (and thus the end consumer) by lowering the average cost of conventional energy procurement.

However, minimizing cost is not the only goal. Another important question is if additional forward markets also lower the *total amount* of conventional energy that a utility company needs to procure. From the environmental viewpoint, this is an extremely relevant question; reducing conventional energy use is one of the key driving factors for the renewable portfolio standards. Indeed, one would desire that the policy decision of adding a forward market for conventional energy increases the efficiency of the available renewable sources by decreasing our consumption of conventional generation.

In order to address the impact of additional markets on total procurement, we study the effect of the addition of a single intermediate (forward) market on the (average) total conventional procurement. Specifically, we compare the average total procurement under the three market scenario with the average total procurement under a scenario where there is no intermediate market. Let us denote the average total procurement under the three market scenario by

$TP_{l,i,r}$, and let $TP_{l,r}$ represent the total procurement under the scenario with no intermediate market.

Intuitively, one expects that the addition of an intermediate market helps utility companies exploit the refined wind estimate to better match its residual demand with the overall conventional procurement. This would suggest that

$$\mathbb{E}[TP_{l,i,r}] < \mathbb{E}[TP_{l,r}];$$

in other words, the addition of an intermediate market helps reduce total procurement. However, as the theorem below suggests, this intuition is not always correct. In fact, for some forecast evolution models and price structures, the addition of an intermediate forward market actually increases the average total procurement.

THEOREM 4. *If the distribution of $\mathcal{E}_2$ has infinite support on the left (i.e., $L_2 = -\infty$), and*

$$\lim_{x \to -\infty} \frac{f_{\mathcal{E}_2}(x)}{F_{\mathcal{E}_2}(x)} = 0, \tag{17}$$

then, there exist values of demand d and deterministic prices satisfying $0 < p_{lt} < p_{in} < p_{rt}$ such that $\mathbb{E}[TP_{l,i,r}] > \mathbb{E}[TP_{l,r}]$.

This theorem provides a counter example to the commonly held assumption that adding forward markets is always beneficial both from the cost and procurement viewpoints. Importantly, Theorem 4 highlights that the total procurement with three markets can be larger than with two markets even in situations with *no price volatility*, i.e., deterministic prices. Thus, the counterintuitive behavior does not result from uncertainty in prices, and can be seen as a consequence of uncertainty about wind generation.

Additionally, it is important to note that the condition given in Equation (17) depends on the probability that the random variable $\mathcal{E}_2$ takes large negative values. Recall from Equation (1) that $\mathcal{E}_2 = \hat{w}_{in} - w$; in other words the condition in the theorem looks at the probability that the wind forecast at the intermediate time $\hat{w}_{in}$ severely *underestimates* the actual wind generation.

We can interpret the condition in Equation (17) as a hazard rate condition. Note that if the distribution of $\mathcal{E}_2$ is symmetric, then the condition can be rewritten as $\lim_{x\to\infty} \frac{f_{\mathcal{E}_2}(x)}{\bar{F}_{\mathcal{E}_2}(x)} = 0$. The quantity in the above equation is the conventionally defined hazard rate function. Condition (17) states that the distribution of $\mathcal{E}_2$ is long-tailed to the left [13, 32]. Long-tailed distributions are an important sub-class of the class of heavy-tailed distributions, and are typically used to model phenomenon in which extreme values occur with a non-negligible probability. Examples of long-tailed distributions include distributions with power-law tails. Therefore, Theorem 4 can be interpreted informally as follows. If the wind estimate at the point of additional markets can significantly underestimate the wind generation w with a non-negligible probability, then the average total procurement may increase with the additional market.

Further, note that the assumption of infinite support of $\mathcal{E}_2$ to the left in the statement of Theorem 4 is not restrictive. Indeed, it follows from Theorem 4 that the inequality $\mathbb{E}[TP_{l,i,r}] > \mathbb{E}[TP_{l,r}]$ can also hold with bounded $\mathcal{E}_2$. Starting with a model instance with infinite support $\mathcal{E}_2$ that satisfies the above inequality, it is easy to show that the inequality would continue to hold if the random variable $\mathcal{E}_2$ is truncated at a large enough value.

Theorem 4 highlights the importance of building better wind forecast models and understanding the distributions of wind forecast errors. The theorem also suggests that, if the probability that the wind forecast severely underestimates the actual available wind is small, then adding an intermediate market might reduce the average total procurement. The following theorem formalizes this intuition by giving a sufficient condition for this to occur. In order to provide a clear comparison with Theorem 4, the sufficient condition is also stated in the context of deterministic prices.

THEOREM 5. *Suppose prices are deterministic and satisfy $0 < p_{lt} < p_{in} < p_{rt}$, and that the distribution of $\mathcal{E}_2$ satisfies the following properties:*

(i) $f_{\mathcal{E}_2}(x)$ is non-decreasing for $x < 0$ and non-increasing for $x > 0$,

(ii) $\frac{f_{\mathcal{E}_2}(x)}{F_{\mathcal{E}_2}(x)}$ is strictly decreasing over $x \in (L_2, 0)$,

(iii) $\frac{f'_{\mathcal{E}_2}(x)}{f_{\mathcal{E}_2}(x)}$ is non-increasing over $x \in (L_2, 0)$.

Then, under Assumption 1, $\mathbb{E}[TP_{l,i,r}] < \mathbb{E}[TP_{l,r}]$.

Theorem 5 highlights that there are a broad set of forecast error distributions for which the introduction of an intermediate market will lower the average total procurement. These are defined via three conditions. The first condition simply states that the density function $f_{\mathcal{E}_2}$ is unimodal at $x = 0$, i.e., it has its maximum at $x = 0$, and is monotone over $x < 0$ and $x > 0$. The last two conditions are concerned with the behavior of the distribution of $\mathcal{E}_2$ when $\mathcal{E}_2 \leq 0$, i.e., when $\hat{w}_{in}$ under estimates the actual wind generation w. Condition *(ii)* is in contrast with the condition in Theorem 4; it requires that the hazard rate of the left tail of $\mathcal{E}_2$ be strictly decreasing. This implies that $\mathcal{E}_2$ is light-tailed to the left. The conditions of Theorem 5 are satisfied by many common distributions, such as the zero mean Gaussian distribution and the zero mean uniform distribution.

Interestingly, it is typical to assume in analytical work that forecast errors are Gaussian [18, 38, 28] (leading to the conclusion of Theorem 5), whereas empirical work on wind power generation suggests that a truncated version of a heavy-tailed Weibull distribution may be a more accurate description [19, 6] (potentially leading to the conclusion of Theorem 4).

Theorems 4 and 5 are proven in the appendix.

7. RELATED LITERATURE

There has been significant recent interest in understanding the impact of the integration of renewable sources into electricity markets. Various aspects of this issue have been studied, including energy storage (see, for example, [3, 21]), demand response (see, for example, [23, 24]), and transmission constraints (see, for example, [40, 11]). In this paper, our focus is on the impact of long term contracts for renewable generation on the efficiency of energy procurement by utility companies.

The problem of energy procurement at the level of the utility company in the presence of renewable generation has been analyzed in recent years by [25, 37, 28, 31], among others. In particular, [25] analyzes a competitive electricity market equilibrium between a generator and a utility company in the presence of wind generation. In contrast, [37, 28, 31] assume that the utility company is a price taker, and pose the procurement problem for the utility company as an optimization with exogenous prices; this is also the approach taken in the present paper. The procurement formulations in [37, 28] assume fixed prices, whereas the formulation in [31] allows for random prices that may be correlated with the wind forecast evolution. Indeed, the procurement formulation (P) in this paper is closely related to the formulations in these papers. However, while the focus of the above mentioned papers is to obtain the optimal procurement strategy for the utility company, the focus of the present paper is to use the optimal strategy as a starting point to investigate the impact of increasing wind penetration and market structure (i.e., placement and inclusion/exclusion of markets).

Another stream of literature that is related to this paper is that related to the classic newsvendor problem [27, 20].

Our optimal procurement formulation (P) is mathematically equivalent to a variant of the newsvendor problem. Indeed, by substituting demand by the residual demand, i.e., demand minus wind (thereby shifting the uncertainty from the supply side to the demand side) and interpreting the real time price as a shortfall penalty, the optimization (P) reduces to single period newsvendor problem in which procurements can be made at two different lead times. See [33] for a survey of the literature relating to such variations of the classic newsvendor problem. However, in contrast to this body of work, the focus of the current paper is to understand the impact on electricity procurement of the *increasing* supply side uncertainty induced by an increasing penetration of renewable resources. To the best of our knowledge, an effort to understand the effect of "scaling up the uncertainty" has not been attempted in the literature pertaining to the newsvendor problem, and so this paper represents a contribution to the newsvendor literature as well.

8. CONCLUDING REMARKS

Our goal in this paper is to quantify the impact of increasing penetration of intermittent, unpredictable, renewable energy on the procurement of conventional generation by utilities, and to understand the impact of proposed changes to the structure of electricity markets. To accomplish this, we consider a three tiered model of electricity procurement in the presence of uncertain renewable generation and price volatility. Within this context, we derive the optimal strategy for procurement of conventional generation and analyze the changes to this strategy as the penetration of renewable increases. Additionally, we study the impact of the addition and placement of an intermediate electricity market on procurement strategies.

The main messages that follow from our results are the following. First, as is commonly recognized, reducing the dependence between the renewable sources (by locating different renewable sources geographically far apart) is key to ensuring efficient utilization of renewable generation. More surprisingly, it turns out that the optimal placement of the intermediate market does not change as renewable penetration increases. Further, the impact of the intermediate market on the total procurement can be either positive or negative depending on the the distribution of forecast errors. The results in this paper highlight that there are many important, and counterintuitive issues with respect to the incorporation of renewable generation into electricity markets.

This work extends easily along several directions. The model in our work can be easily extended to allow for an arbitrary number of forward markets for conventional energy procurement. Another extension that can be easily incorporated in our model is to allow utility companies to sell excess conventional generation in forward markets (as in [28]). In our model, we assumed that the exact realized wind energy is known at the time of the real time market. This assumption can also be relaxed by introducing a third distribution that reflects the error between wind forecasts in the real time market and the actual realized wind (as in [28]). The results presented in this paper can be easily generalized to this setting under the requirement that the utility company maintains a certain loss of load probability at the time of the real time market.

There are also, however, some non-trivial extensions to this model which are interesting opportunities for future research. One such extension is to understand the role of large volumes of grid energy storage on the optimal procurement policy of utility companies. Another extension of interest is to take into account ramp constraints of various generators, which would couple procurement decisions between different time instants.

More generally, the big question for the area is how and where renewables should be allowed to interact with electricity markets. The setting considered here is one extreme, where renewable energy only interacts through long-term contracts. The other extreme, where renewable energy interacts only in the real time market, has also been studied in recent work (for example, see [5]). Hopefully, building on these results, we can move toward a unified approach highlighting the most efficient manner for renewable energy to interact with the electricity markets.

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APPENDIX

In this section, we prove Theorem 4 and Theorem 5. We first characterize the optimal procurement policy with deterministic prices in Section A. We then prove some preliminary lemmas in Section B. In Section C, we prove Theorem 5. Finally, we prove Theorem 4 in Section D. It is important to note that throughout this section, we consider deterministic prices, satisfying $0 < p_{lt} < p_{in} < p_{rt}$.

A. OPTIMAL PROCUREMENT POLICY WITH DETERMINISTIC PRICES

In this section, we first analyze the optimal procurement strategy for the utility company in the three tiered market scenario with deterministic prices.

LEMMA 2. *The optimal procurement strategy for the utility company in the three tiered market scenario with deterministic prices satisfying $0 < p_{lt} < p_{in} < p_{rt}$ is the following. $q_{lt}^* = [d - \hat{w}_{lt} + r_{lt}]_+$, $q_{in}^* = [d - \hat{w}_{in} - q_{lt} + r_{in}]_+$, and $q_{rt}^* = [d - w - (q_{lt} + q_{in})]_+$. Here, $r_{in} = \bar{F}_{\mathcal{E}_2}^{-1}\left(\frac{p_{in}}{p_{rt}}\right)$, and r_{lt} is the unique solution of*

$$p_{lt} - p_{in}\bar{F}_{\mathcal{E}_1}(r - r_{in}) - p_{rt}P\left(\mathcal{E}_1 + \mathcal{E}_2 > r, \mathcal{E}_1 \leq r - r_{in}\right) = 0. \tag{18}$$

Next, in order to understand the effect of the addition of a market, we study the case where there is no intermediate market.

LEMMA 3. *The optimal procurement strategy for the utility company in the two tier market scenario with no intermediate market, and with deterministic prices satisfying $0 < p_{lt} < p_{rt}$ is the following.*

$$q_{lt}^* = [d - \hat{w}_{lt} + \bar{r}_{lt}]_+\,,\ q_{rt}^* = [d - q_{lt} - w]_+\,,$$

where $\bar{r}_{lt} := \bar{F}_{\mathcal{E}_1+\mathcal{E}_2}^{-1}\left(\frac{p_{lt}}{p_{rt}}\right)$.

Lemmas 2 and 3 can be proved easily along the same lines as Theorem 1 and the proofs are omitted.

B. PRELIMINARY LEMMAS

In the next two lemmas, we study the sensitivity of the optimal reserves and procurement with respect to the intermediate price p_{in} (recall that the prices are deterministic). For the remainder of this section, we keep the distributions $\mathcal{E}_1$ and $\mathcal{E}_2$, as well as the prices p_{lt} and p_{rt} fixed. Define $\bar{r}_{in} := \bar{F}_{\mathcal{E}_2}^{-1}\left(\frac{p_{lt}}{p_{rt}}\right)$. We can think of $\bar{r}_{in}$ as the optimal reserve in a two market scenario, where the long term market occurs at time $-T_{in}$.

LEMMA 4. *As p_{in} is varied over (p_{lt}, p_{rt}), the following results hold.*

1. *The intermediate reserve r_{in} is a strictly decreasing function of the intermediate price p_{in} with*
$$\frac{\partial r_{in}}{\partial p_{in}} = -\frac{1}{p_{rt} f_{\mathcal{E}_2}(r_{in})}. \tag{19}$$
Furthermore,
$$\lim_{p_{in}\downarrow p_{lt}} r_{in} = \bar{r}_{in}, \qquad \lim_{p_{in}\uparrow p_{rt}} r_{in} = L_2. \tag{20}$$
2. *The long term reserve r_{lt} is a non-decreasing function of the intermediate price p_{in} with*
$$\frac{\partial r_{lt}}{\partial p_{in}} = \frac{\bar{F}_{\mathcal{E}_1}(r_{lt} - r_{in})}{p_{rt}\int_{-\infty}^{r_{lt}-r_{in}} f_{\mathcal{E}_1}(y) f_{\mathcal{E}_2}(r_{lt} - y)dy}. \tag{21}$$
Furthermore,
$$\lim_{p_{in}\downarrow p_{lt}} r_{lt} = L_1 + \bar{r}_{in}, \qquad \lim_{p_{in}\uparrow p_{rt}} r_{lt} = \bar{r}_{lt} = \bar{F}_{\mathcal{E}_1+\mathcal{E}_2}^{-1}\left(\frac{p_{lt}}{p_{rt}}\right) \tag{22}$$

PROOF. Equations (19) and (21) follow from an application of the implicit function theorem [29]. The statements in (20) follow easily from the definition of r_{in} in Lemma 2, noting that $[L_2, R_2]$ denotes the support of the random variable $\mathcal{E}_2$.

Note that r_{lt} is a monotone function of the intermediate price p_{lt}. Hence, as $p_{in} \downarrow p_{lt}$ or $p_{in} \uparrow p_{rt}$, the long term reserve r_{lt} has a limit. To prove the first statement of (22), suppose that as $p_{in} \downarrow p_{lt}$, $r_{lt} \to \check{r}_{lt}$. From Equation (18), (by taking the limit $p_{in} \downarrow p_{lt}$), we note that $\check{r}_{lt}$ is a solution of the equation

$$\varphi(r) := p_{lt} - p_{lt}\bar{F}_{\mathcal{E}_1}(r - \bar{r}_{in}) - p_{rt}\int_{z=r}^{\infty}\int_{y=-\infty}^{r-\bar{r}_{in}} f_{\mathcal{E}_1}(y) f_{\mathcal{E}_2}(z-y)dzdy = 0.$$

It is easy to check that (i) $\varphi(r) = 0$ for $r \le L_1 + \bar{r}_{in}$; (ii) for $r \in (L_1 + \bar{r}_{in}, R_1 + R_2)$, we have $\varphi'(r) = p_{rt}\int_{r_{in}}^{\infty} f_{\mathcal{E}_1}(r - z)f_{\mathcal{E}_2}(z)dz > 0$; and (iii) for $r > R_1 + R_2$, $\varphi'(r) = 0$. Thus we conclude that $\varphi(r) > 0$ for $r > L_1 + \bar{r}_{in}$, which implies that $\check{r}_{lt} \le L_1 + \bar{r}_{in}$. However, for any $p_{in} \in (p_{lt}, p_{rt})$, we have $r_{lt} > L_1 + r_{in}$, which implies that $\check{r}_{lt} \ge L_1 + \bar{r}_{in}$. Therefore, $\check{r}_{lt} = L_1 + \bar{r}_{in}$, which proves the first statement of (22).

To prove the second statement of (22), suppose that as $p_{in} \uparrow p_{rt}$, $r_{lt} \to \hat{r}_{lt}$. Taking the limit as $p_{in} \uparrow p_{rt}$ in Equation (18), we conclude that $\hat{r}_{lt}$ satisfies

$$p_{lt} - p_{rt}P\left(\mathcal{E}_1 > \hat{r}_{lt} - L_2\right) - p_{rt}P\left(\mathcal{E}_1 + \mathcal{E}_2 > \hat{r}_{lt}; \mathcal{E}_1 \le \hat{r}_{lt} - L_2\right) = 0.$$

Define A to be the event that $\{\mathcal{E}_1 + \mathcal{E}_2 > \hat{r}_{lt}\}$ and B to be the event that $\{\mathcal{E}_1 > \hat{r}_{lt} - L_2\}$. Using the fact that $P(A \cap B^c) + P(B) = P(A \cup B)$, we get that $\hat{r}_{lt}$ satisfies the equation

$$p_{lt} - p_{rt}P\left(\{\mathcal{E}_1 + \mathcal{E}_2 > \hat{r}_{lt}\} \cup \{\mathcal{E}_1 > \hat{r}_{lt} - L_2\}\right) = 0.$$

Note that the random variable $\mathcal{E}_2 \in (L_2, R_2)$ almost surely, and hence the event $\{\mathcal{E}_1 > \hat{r}_{lt} - L_2\}$ implies the event $\{\mathcal{E}_1 + \mathcal{E}_2 > \hat{r}_{lt}\}$ almost surely. Thus, the long term reserve $\hat{r}_{lt}$ satisfies the equation

$$p_{lt} - p_{rt}P\left(\mathcal{E}_1 + \mathcal{E}_2 > \hat{r}_{lt}\right) = 0,$$

which implies the second statement of (22). This completes the proof. . $\square$

The next lemma studies how the total procurement changes as a function of the intermediate price p_{in}. Recall that $\mathbb{E}[TP_{l,i,r}]$ and $\mathbb{E}[TP_{l,r}]$ denote, respectively, the expected total procurement under the three market scenario (long term, intermediate, real time) and the two market scenario (long term, real time).

For the remainder of this section, we introduce some additional notation. Let $q^*_{lt;l,i,r}$, $q^*_{in;l,i,r}$, and $q^*_{rt;l,i,r}$ denote the *optimal* procurements in the long term, intermediate, and the real time market under the scenario that the utility company has three opportunities to procure energy. These optimal quantities are given in Lemma 2. Similarly, we let $q^*_{lt;l,r}$ and $q^*_{rt;l,r}$ to be the optimal procurements in the long term and in the real time under the scenario where there is no intermediate market. These quantities are given by Lemma 3.

Lemma 5. *As the intermediate price p_{in} approaches the real time price p_{rt}, the expected total procurement in three market scenario converges to the expected total procurement in the two market scenario. That is, $\lim_{p_{in}\uparrow p_{rt}} \mathbb{E}[TP_{l,i,r}] = \mathbb{E}[TP_{l,r}]$.*

Proof. We prove this lemma using a sample path argument. A sample path is defined by the values of $\hat{w}_{lt}$, $\mathcal{E}_1$, and $\mathcal{E}_2$. We will show that as $p_{in} \uparrow p_{rt}$,

$$q^*_{lt;l,i,r} + q^*_{in;l,i,r} + q^*_{rt;l,i,r} \longrightarrow q^*_{lt;l,r} + q^*_{rt;l,r} \tag{23}$$

almost surely. An elementary application of the dominated convergence theorem would then prove the lemma.

From Lemmas 2, 3, and 4, it is easy to show that as $p_{in} \uparrow p_{rt}$, we have

$$q^*_{lt;l,i,r} \longrightarrow q^*_{lt;l,r} \tag{24}$$

almost surely. Thus, to prove (23), it suffices to show that

$$q^*_{in;l,i,r} + q^*_{rt;l,i,r} \longrightarrow q^*_{rt;l,r}, \tag{25}$$

almost surely as $p_{in} \uparrow p_{rt}$. Recall that $q^*_{rt;l,r} = \left[d - w - q^*_{lt;l,r}\right]_+$ From Lemma 4 and Equation (24), we conclude that as $p_{in} \uparrow p_{rt}$. we have

$$\begin{array}{lll} q^*_{in;l,i,r} & \longrightarrow & \left[d - \hat{w}_{in} - q^*_{lt;l,r} + L_2\right]_+, \\ q^*_{rt;l,i,r} & \longrightarrow & \left[d - w - q^*_{lt;l,r} - [d - \hat{w}_{in} + L_2]_+\right]_+. \end{array} \tag{26}$$

To see that Equation (26) implies (25), we consider the following two cases. On sample paths satisfying $d < \hat{w}_{in} + q^*_{lt;l,r} - L_2$, we have

$$q^*_{in;l,i,r} \xrightarrow{p_{in}\uparrow p_{rt}} 0, \quad q^*_{rt;l,i,r} \xrightarrow{p_{in}\uparrow p_{rt}} \left[d - w - q^*_{lt;l,r}\right]_+,$$

which implies Equation (25). On sample paths satisfying $d \ge \hat{w}_{in} + q^*_{lt;l,r} - L_2$, we have

$$q^*_{in;l,i,r} \xrightarrow{p_{in}\uparrow p_{rt}} d - \hat{w}_{in} - q^*_{lt;l,r} + L_2, \; q^*_{rt;l,i,r} \xrightarrow{p_{in}\uparrow p_{rt}} \left[\mathcal{E}_2 - L_2\right]_+.$$

Noting that $\mathcal{E}_2 \ge L_2$ almost surely, we conclude that except possibly on a measure zero set of sample paths, we have

$$q^*_{in;l,i,r} + q^*_{rt;l,i,r} \xrightarrow{p_{in}\uparrow p_{rt}} (d - \hat{w}_{in} - q^*_{lt;l,r} + L_2) + (\mathcal{E}_2 - L_2) = d - w - q^*_{lt;l,r}.$$

Since the procurements are always non-negative, the above equation implies Equation (25). This completes the proof. $\square$

Next, we prove the following lemma, which is used in the proof of Theorem 5.

Lemma 6. *Suppose that the distribution of $\mathcal{E}_2$ satisfies the conditions (i), (ii), and (iii) of Theorem 5. Then*

$$\left(\frac{p_{rt} - p_{lt}}{p_{rt} - p_{in}}\right) f_{\mathcal{E}_2}(r_{in}) > \int_{-\infty}^{r_{lt}-r_{in}} f_{\mathcal{E}_1}(y) f_{\mathcal{E}_2}(r_{lt} - y)dy + \bar{F}_{\mathcal{E}_1}(r_{lt} - r_{in}) f_{\mathcal{E}_2}(r_{in}). \tag{27}$$

Proof. For this proof, we use the notation $\beta := \bar{F}_{\mathcal{E}_2}(0)$. Let us also denote by $L(p_{in})$ and $R(p_{in})$ the left hand side and the right hand side of the Inequality (27) respectively. To prove the lemma, we need to show that $L(p_{in}) > R(p_{in})$ for all values of $p_{in} \in (p_{lt}, p_{rt})$. Let us consider two separate cases.

Case 1: $p_{lt} < \beta p_{rt}$.

Consider first the case $p_{in} \in (p_{lt}, \beta p_{rt}]$. For this case, we have $p_{in}/p_{rt} \le \beta$ and hence from Lemma 2, we have that $r_{in} \ge 0$. Using a change of variables, we can write $R(p_{in})$ as

$$R(p_{in}) = \int_{r_{in}}^{\infty} f_{\mathcal{E}_1}(r_{lt} - y) f_{\mathcal{E}_2}(y)\, dy + \bar{F}_{\mathcal{E}_1}(r_{lt} - r_{in}) f_{\mathcal{E}_2}(r_{in}). \tag{28}$$

Note that $f_{\mathcal{E}_2}(x)$ is non increasing for all $x > 0$. Thus, for all $y \ge r_{in} \ge 0$, we have $f_{\mathcal{E}_2}(r_{in}) \ge f_{\mathcal{E}_2}(y)$. Therefore,

$$R(p_{in}) \le f_{\mathcal{E}_2}(r_{in}) \int_{r_{in}}^{\infty} f_{\mathcal{E}_1}(r_{lt} - y)\, dy + \bar{F}_{\mathcal{E}_1}(r_{lt} - r_{in}) f_{\mathcal{E}_2}(r_{in}) = f_{\mathcal{E}_2}(r_{in}) < L(p_{in}),$$

where the last inequality follows from that fact that $p_{lt} < p_{in}$. Next, consider the case where $p_{in} \in (\beta p_{rt}, p_{rt})$. From Lemma 2, we have that $r_{in} < 0$. Also, from the conditions of the lemma, we have that $f_{\mathcal{E}_2}(y) \le f_{\mathcal{E}_2}(0)$ for all $y \in \mathbb{R}$.

Thus, using the representation (28), we have

$$R(p_{in}) \leq f_{\mathcal{E}_2}(0) \int_{r_{in}}^{\infty} f_{\mathcal{E}_1}(r_{lt}-y)\ dy + \bar{F}_{\mathcal{E}_1}(r_{lt}-r_{in}) f_{\mathcal{E}_2}(0)$$
$$= f_{\mathcal{E}_2}(0). \tag{29}$$

Let us now look at $L(p_{in})$. We will construct a lower bound for $L(p_{in})$ for $p_{in} \in (\beta p_{rt}, p_{rt})$. Note that $\bar{F}_{\mathcal{E}_2}(r_{in}) = p_{in}/p_{rt}$ and hence we can write $L(p_{in})$ as $L(p_{in}) = \left(\frac{p_{rt}-p_{lt}}{p_{rt}}\right) \frac{f_{\mathcal{E}_2}(r_{in})}{F_{\mathcal{E}_2}(r_{in})}$. Recall that for $p_{in} \in (\beta p_{rt}, p_{rt})$, we have $r_{in} < 0$. Furthermore from Lemma 4, r_{in} is strictly decreasing function of p_{in}. From the statement of the lemma, $f_{\mathcal{E}_2}(x)/F_{\mathcal{E}_2}(x)$ is a strictly decreasing function for all $x \leq 0$. Hence, $L(p_{in})$ is strictly increasing over $p_{in} \in (\beta p_{rt}, p_{rt})$. Thus, we have

$$L(p_{in}) > L(\beta p_{rt}) = \left(\frac{p_{rt}-p_{lt}}{p_{rt}-\beta p_{rt}}\right) f_{\mathcal{E}_2}(0) > f_{\mathcal{E}_2}(0),$$

where we have used the fact that $r_{in} = 0$ when $p_{in} = \beta p_{rt}$, and $p_{lt} < \beta p_{rt}$. The above inequality, combined with Condition (29), implies that $L(p_{in}) > R(p_{in})$. This proves Case 1.

Case 2: $p_{lt} \geq \beta p_{rt}$.

In this case, we note that $p_{in}/p_{rt} > \beta$ and hence from Lemma 2, we get that $r_{in} < 0$. Using an argument similar to Case 1, it is easy to verify that $L(p_{in})$ is a strictly increasing function for $p_{in} \in (p_{lt}, p_{rt})$. Therefore,

$$L(p_{in}) > \lim_{p_{in} \downarrow p_{lt}} L(p_{in}) = f_{\mathcal{E}_2}(\bar{r}_{in}),$$

where we use the fact that as $p_{in} \downarrow p_{lt}$, $r_{in} \rightarrow \bar{r}_{in}$ (Lemma 4). Thus, to prove the lemma for this case, it suffices to show that $R(p_{in}) \leq f_{\mathcal{E}_2}(\bar{r}_{in})$. From Lemma 4, we note that

$$\lim_{p_{in} \downarrow p_{lt}} r_{lt} - r_{in} = L_1$$

and hence

$$\lim_{p_{in} \downarrow p_{lt}} R(p_{in}) = f_{\mathcal{E}_2}(\bar{r}_{in}).$$

Thus, to prove the lemma, it suffices to show that $R(p_{in})$ is non-increasing over the range $p_{in} \in (p_{lt}, p_{rt})$. To show this, we note that

$$\begin{aligned}
&\frac{\partial R(p_{in})}{\partial p_{in}} \\
&= \frac{\partial}{\partial p_{in}} \Bigg(\int_{-\infty}^{r_{lt}-r_{in}} f_{\mathcal{E}_1}(y) f_{\mathcal{E}_2}(r_{lt}-y) dy \\
&\qquad + \bar{F}_{\mathcal{E}_1}(r_{lt}-r_{in}) f_{\mathcal{E}_2}(r_{in}) \Bigg) \\
&= \frac{\partial r_{lt}}{\partial r_{in}} \int_{-\infty}^{r_{lt}-r_{in}} f_{\mathcal{E}_1}(y) f'_{\mathcal{E}_2}(r_{lt}-y) dy \\
&\qquad + \frac{\partial r_{in}}{\partial p_{in}} \bar{F}_{\mathcal{E}_1}(r_{lt}-r_{in}) f'_{\mathcal{E}_2}(r_{in}) \\
&= \frac{\bar{F}_{\mathcal{E}_1}(r_{lt}-r_{in})}{p_{rt}} \left[\frac{\int_{-\infty}^{r_{lt}-r_{in}} f_{\mathcal{E}_1}(y) f'_{\mathcal{E}_2}(r_{lt}-y) dy}{\int_{-\infty}^{r_{lt}-r_{in}} f_{\mathcal{E}_1}(y) f_{\mathcal{E}_2}(r_{lt}-y) dy} - \frac{f'_{\mathcal{E}_2}(r_{in})}{f_{\mathcal{E}_2}(r_{in})} \right] \\
&= \frac{\bar{F}_{\mathcal{E}_1}(r_{lt}-r_{in})}{p_{rt}} \left[\frac{\int_{r_{in}}^{\infty} f_{\mathcal{E}_1}(r_{lt}-y) f'_{\mathcal{E}_2}(y) dy}{\int_{r_{in}}^{\infty} f_{\mathcal{E}_1}(r_{lt}-y) f_{\mathcal{E}_2}(y) dy} - \frac{f'_{\mathcal{E}_2}(r_{in})}{f_{\mathcal{E}_2}(r_{in})} \right].
\end{aligned} \tag{30}$$

Here the third equality follows from Lemma 4 and the last equality follows from a change of variable. We will show that $\frac{\partial R(p_{in})}{\partial p_{in}} \leq 0$ by considering the following two separate cases.

Case 2a: Consider the case $\int_{r_{in}}^{0} f_{\mathcal{E}_1}(r_{lt}-y) f_{\mathcal{E}_2}(y) dy = 0$. Recall that the density functions $f_{\mathcal{E}_1}(\cdot)$ and $f_{\mathcal{E}_2}(\cdot)$ are assumed to be strictly positive over the interior of the supports of $\mathcal{E}_1$ and $\mathcal{E}_2$ respectively. Therefore, under Case 2a, the intersection of the supports of the functions $f_{\mathcal{E}_2}(y)$ and $f_{\mathcal{E}_1}(r_{lt}-y)$ and $[r_{in}, 0]$ has measure zero. This implies that

$$\int_{r_{in}}^{0} f_{\mathcal{E}_1}(r_{lt}-y) f'_{\mathcal{E}_2}(y) dy = 0. \tag{31}$$

Using the condition defining Case 2a and (31), the expression for $\frac{\partial R(p_{in})}{\partial p_{in}}$ in (30) can be rewritten as

$$\frac{\bar{F}_{\mathcal{E}_1}(r_{lt}-r_{in})}{p_{rt}} \left[\frac{\int_{0}^{\infty} f_{\mathcal{E}_1}(r_{lt}-y) f'_{\mathcal{E}_2}(y) dy}{\int_{0}^{\infty} f_{\mathcal{E}_1}(r_{lt}-y) f_{\mathcal{E}_2}(y) dy} - \frac{f'_{\mathcal{E}_2}(r_{in})}{f_{\mathcal{E}_2}(r_{in})} \right] \leq 0.$$

The inequality above follows from the assumption that $f'_{\mathcal{E}_2}(x) \leq 0$ for $x \geq 0$.

Case 2b: Consider the case $\int_{r_{in}}^{0} f_{\mathcal{E}_1}(r_{lt}-y) f_{\mathcal{E}_2}(y) dy > 0$. Since $f'_{\mathcal{E}_2}(x) \leq 0$ for $x \geq 0$,

$$\int_{r_{in}}^{\infty} f_{\mathcal{E}_1}(r_{lt}-y) f'_{\mathcal{E}_2}(y) dy \leq \int_{r_{in}}^{0} f_{\mathcal{E}_1}(r_{lt}-y) f'_{\mathcal{E}_2}(y) dy.$$

Therefore, the expression for $\frac{\partial R(p_{in})}{\partial p_{in}}$ in (30) can be bounded by

$$\frac{\bar{F}_{\mathcal{E}_1}(r_{lt}-r_{in})}{p_{rt}} \left[\frac{\int_{r_{in}}^{0} f_{\mathcal{E}_1}(r_{lt}-y) f'_{\mathcal{E}_2}(y) dy}{\int_{r_{in}}^{0} f_{\mathcal{E}_1}(r_{lt}-y) f_{\mathcal{E}_2}(y) dy} - \frac{f'_{\mathcal{E}_2}(r_{in})}{f_{\mathcal{E}_2}(r_{in})} \right].$$

Define

$$p(y) = \frac{f_{\mathcal{E}_1}(r_{lt}-y)}{\int_{r_{in}}^{0} f_{\mathcal{E}_1}(r_{lt}-y)\ dy},\ \xi(y) = \frac{f'_{\mathcal{E}_2}(y)}{f_{\mathcal{E}_2}(y)},\ \varphi(y) = f_{\mathcal{E}_2}(y).$$

Note that $p(y)$ is a valid probability density function defined over the interval $[r_{in}, 0]$. We can then write

$$\frac{\partial R(p_{in})}{\partial p_{in}} \leq \frac{\bar{F}_{\mathcal{E}_1}(r_{lt}-r_{in})}{p_{rt}} \left[\frac{\int_{r_{in}}^{0} p(y)\xi(y)\varphi(y) dy}{\int_{r_{in}}^{0} p(y)\varphi(y) dy} - \frac{f'_{\mathcal{E}_2}(r_{in})}{f_{\mathcal{E}_2}(r_{in})} \right]$$

Using Lemma 7 below, we get

$$\begin{aligned}
\frac{\partial R(p_{in})}{\partial p_{in}} &\leq \frac{\bar{F}_{\mathcal{E}_1}(r_{lt}-r_{in})}{p_{rt}} \left[\int_{r_{in}}^{0} p(y) \frac{f'_{\mathcal{E}_2}(y)}{f_{\mathcal{E}_2}(y)}\ dy - \frac{f'_{\mathcal{E}_2}(r_{in})}{f_{\mathcal{E}_2}(r_{in})} \right] \\
&\leq 0,
\end{aligned}$$

The last inequality follows from the fact that $f'_{\mathcal{E}_2}(y)/f_{\mathcal{E}_2}(y)$ is non-increasing over the range $y \in (r_{in}, 0)$. We have now proved that $R(p_{in})$ is non-increasing over $p_{in} \in (p_{lt}, p_{rt})$. This completes the proof of Case 2. $\square$

The above lemma uses the following technical lemma.

LEMMA 7. *Let $[a,b]$ denote an interval on the real line. Suppose that $\xi : [a,b] \rightarrow \mathbb{R}$ is non-increasing, and $\varphi : [a,b] \rightarrow \mathbb{R}$ is non-decreasing. Also, suppose that $p(\cdot)$ is a probability density function defined over the interval $[a,b]$. Then* $\int_a^b p(y)\xi(y)\varphi(y)\ dy \leq \left(\int_a^b p(y)\xi(y)\ dy\right)\left(\int_a^b p(y)\varphi(y)\ dy\right)$.

To prove the above lemma, we look at the discrete version of the above inequality. The above lemma then follows naturally by taking a limit of Reimann sums.

LEMMA 8. *Consider finite sequences $\{\xi_i\}_{i=1}^n$, $\{\varphi_i\}_{i=1}^n$, and $\{p_i\}_{i=1}^n$, satisfying (i) φ_i is non-decreasing, (ii) ξ_i is non-increasing, and (iii) $p_i \geq 0$, $\sum_{i=1}^n p_i = 1$. Then $\sum_i p_i\xi_i\varphi_i \leq \left(\sum_i p_i\xi_i\right)\left(\sum_i p_i\varphi_i\right)$.*

PROOF. Proof. We can write the right hand side of the above inequality as $\sum_i p_i^2\xi_i\varphi_i + \sum_{i,j:i\neq j} p_ip_j\xi_i\varphi_j$. Thus to prove the lemma, it suffices to show that

$$\begin{aligned}&\sum_i p_i\xi_i\varphi_i \le \sum_i p_i^2\xi_i\varphi_i + \sum_{i,j:i\ne j} p_ip_j\xi_i\varphi_j\\ \iff &\sum_i p_i(1-p_i)\xi_i\varphi_i \le \sum_{i,j:i\ne j} p_ip_j\xi_i\varphi_j\\ \iff &\sum_{i,j:j\ne i} p_ip_j\xi_i\varphi_i \le \sum_{i,j:i\ne j} p_ip_j\xi_i\varphi_j.\end{aligned}$$

The last inequality above uses the fact that $(1-p_i) = \sum_{j\ne i} p_j$. To prove the last inequality above, it suffices to check that for $i \ne j$, $\xi_i\varphi_i + \xi_j\varphi_j \le \xi_i\varphi_j + \xi_j\varphi_i$. This inequality is equivalent to $(\xi_i - \xi_j)(\varphi_i - \varphi_j) \le 0$, which follows from the monotonicity assumptions on the sequences $\{\xi_i\}_{i=1}^n$, $\{\varphi_i\}_{i=1}^n$. This completes the proof. □

C. PROOF OF THEOREM 5

Let p_{lt}, p_{in} and p_{rt} be the (deterministic) prices in the three markets. Let $\Gamma(p)$ be the expected optimal total procurement in three markets, when the long term price is p_{lt}, the real time price is p_{rt}, and the intermediate price is p. In other words, $\Gamma(p)$ gives the expected total procurement for a variable intermediate price (denoted by p). Clearly, we have $\mathbb{E}[TP_{l,i,r}] = \Gamma(p_{in})$. From Lemma 5, we know that $\mathbb{E}[TP_{l,r}] = \lim_{p\uparrow p_{rt}} \Gamma(p)$. Thus, to prove the theorem it suffices to show that $\Gamma(p)$ is a strictly increasing function of p.

From the statement of the theorem, Assumption 1 holds for prices p_{lt}, p_{in} and p_{rt}. From Lemma 4, we know that r_{lt} is a non-decreasing function of intermediate price. Hence for all $p \ge p_{in}$, Assumption 1 holds. Thus, the optimal procurements for all intermediate price $p \ge p_{in}$ are given by Equation 7. Let $q^*_{lt}(p)$, $q^*_{in}(p)$, and $q^*_{rt}(p)$ be the optimal procurements for the intermediate price p. Thus, we have

$$\frac{\partial \mathbb{E}[q^*_{lt}(p)]}{\partial p} = \frac{\partial}{\partial p}(d - \hat{w}_{lt} + r_{lt}) = \frac{\partial r_{lt}}{\partial p}.$$

$$\begin{aligned}\frac{\partial \mathbb{E}[q^*_{in}(p)]}{\partial p} &= \frac{\partial}{\partial p}\mathbb{E}\left[\mathcal{E}_1 - (r_{lt} - r_{in})\right]_+\\ &= \bar{F}_{\mathcal{E}_1}(r_{lt} - r_{in})\left(\frac{\partial r_{in}}{\partial p} - \frac{\partial r_{lt}}{\partial p}\right).\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathbb{E}[q^*_{rt}(p)]}{\partial p} &= \frac{\partial}{\partial p}\mathbb{E}\left[\mathcal{E}_2 - r_{lt} + \min\{\mathcal{E}_1, r_{lt} - r_{in}\}\right]_+\\ &= \frac{\partial}{\partial p}\Bigg(\bar{F}_{\mathcal{E}_1}(r_{lt} - r_{in})\int_{r_{in}}^{\infty}(z - r_{in})f_{\mathcal{E}_2}(z)dz\\ &\quad + \int_{z=r_{lt}}^{\infty}\int_{y=-\infty}^{r_{lt}-r_{in}}(z - r_{lt})f_{\mathcal{E}_1}(y)f_{\mathcal{E}_2}(z-y)dzdy\Bigg)\\ &= -\bar{F}_{\mathcal{E}_1}(r_{lt} - r_{in})\bar{F}_{\mathcal{E}_2}(r_{in})\frac{\partial r_{in}}{\partial p}\\ &\quad - \frac{\partial r_{lt}}{\partial p}\int_{z=r_{lt}}^{\infty}\int_{y=-\infty}^{r_{lt}-r_{in}}(z - r_{lt})f_{\mathcal{E}_1}(y)f_{\mathcal{E}_2}(z-y)dzdy.\end{aligned}$$

The last step above follows from a direct differentiation. Combining the above equations, we have

$$\begin{aligned}\frac{\partial \Gamma(p)}{\partial p} &= \frac{\partial r_{lt}}{\partial p}\Bigg[1 - \bar{F}_{\mathcal{E}_1}(r_{lt} - r_{in})\\ &\qquad - \int_{z=r_{lt}}^{\infty}\int_{y=-\infty}^{r_{lt}-r_{in}}(z - r_{lt})f_{\mathcal{E}_1}(y)f_{\mathcal{E}_2}(z-y)dzdy\Bigg]\\ &\qquad + \frac{\partial r_{in}}{\partial p}\left[\bar{F}_{\mathcal{E}_1}(r_{lt} - r_{in}) - \bar{F}_{\mathcal{E}_1}(r_{lt} - r_{in})\bar{F}_{\mathcal{E}_2}(r_{in})\right]\\ &= \frac{\partial r_{lt}}{\partial p}\left[1 - \frac{p_{lt}}{p_{rt}} - \left(1 - \frac{p}{p_{rt}}\right)\bar{F}_{\mathcal{E}_1}(r_{lt} - r_{in})\right]\\ &\qquad + \frac{\partial r_{in}}{\partial p}\bar{F}_{\mathcal{E}_1}(r_{lt} - r_{in})\left(1 - \frac{p}{p_{rt}}\right).\end{aligned}$$

The last equality follows from the fact that $h(r_{lt}) = 0$ and that $\bar{F}_{\mathcal{E}_2}(r_{in}) = p/p_{rt}$. Substituting $\partial r_{lt}/\partial p$ and $\partial r_{in}/\partial p$ from Lemma 4 and after some algebraic manipulation, we get that $\partial\Gamma(p)/\partial p > 0$ if and only if

$$\begin{aligned}&\left(\frac{p_{rt} - p_{lt}}{p_{rt} - p}\right)f_{\mathcal{E}_2}(r_{in}) >\\ &\qquad \int_{-\infty}^{r_{lt}-r_{in}} f_{\mathcal{E}_1}(y)f_{\mathcal{E}_2}(r_{lt} - y)dy + \bar{F}_{\mathcal{E}_1}(r_{lt} - r_{in})f_{\mathcal{E}_2}(r_{in}).\end{aligned}$$

From Lemma 6, we know that this inequality holds for all $p \in [p_{in}, p_{rt})$. This completes the proof.

D. PROOF OF THEOREM 4

Let us fix some long term and real time market prices p_{lt} and p_{rt} such that $0 < p_{lt} < p_{rt}$. Let $\Gamma(p)$ be the expected optimal total procurement in a three market scenario with long term price p_{lt}, real time price p_{rt}, and the intermediate price p. From Lemma 5, we know that $\lim_{p\uparrow p_{rt}} \Gamma(p) = \mathbb{E}[TP_{l,r}]$. So to prove the lemma, it suffices to show that there exists $p_{in} \in (p_{lt}, p_{rt})$ such that for all intermediate market prices $p \in (p_{in}, p_{rt})$, $\frac{\partial\Gamma(p)}{\partial p} < 0$. Indeed, this would imply that $\Gamma(p_{in}) > \lim_{p\uparrow p_{rt}} \Gamma(p) = \mathbb{E}[TP_{l,r}]$. Since $\Gamma(p_{in}) = \mathbb{E}[TP_{l,i,r}]$, the expected total procurement in three market scenario under the intermediate price p_{in}, this implies a tuple of prices satisfying the claim of the lemma.

Pick $\bar{p} \in (p_{lt}, p_{rt})$. Now choose demand d large enough such that Assumption 1 holds when the intermediate price $p = \bar{p}$. From Lemma 4, we know that r_{lt} is a non decreasing function of intermediate price p. Thus, Assumption 1 continues to hold for all intermediate price $p \ge \bar{p}$. Following the argument similar to the proof of Theorem 5, we can show that for all $p \in [\bar{p}, p_{rt})$, $\frac{\partial\Gamma(p)}{\partial p} < 0$ if and only if

$$\begin{aligned}&\left(\frac{p_{rt} - p_{lt}}{p_{rt} - p}\right)f_{\mathcal{E}_2}(r_{in}) <\\ &\qquad \int_{-\infty}^{r_{lt}-r_{in}} f_{\mathcal{E}_1}(y)f_{\mathcal{E}_2}(r_{lt} - y)dy + \bar{F}_{\mathcal{E}_1}(r_{lt} - r_{in})f_{\mathcal{E}_2}(r_{in}).\end{aligned} \tag{32}$$

Let us denote by $L(p)$ and $R(p)$ the left hand side and the right hand side of the above inequality respectively.

From the definition of r_{in} (Lemma 2),

$$L(p) = \left(\frac{p_{rt} - p_{lt}}{p_{rt}}\right)\frac{f_{\mathcal{E}_2}(r_{in})}{F_{\mathcal{E}_2}(r_{in})}.$$

Since $\lim_{p\uparrow p_{rt}} r_{in} = -\infty$ (Lemma 4), the conditions of the theorem imply that $\lim_{p\uparrow p_{rt}} L(p) = 0$.

Now let us look at $R(p)$. From Lemma 4, note that

$$\lim_{p\uparrow p_{rt}}(r_{lt} - r_{in}) = \infty, \quad \lim_{p\uparrow p_{rt}} r_{lt} = \bar{r}_{lt}.$$

Thus, using the dominated convergence theorem, we get that

$$\lim_{p \uparrow p_{rt}} R(p) = f_{\mathcal{E}_1+\mathcal{E}_2}(\bar{r}_{lt}) > 0.$$

The above statements imply that there exists $p_{in} \in (\bar{p}, p_{rt})$ such that for all $p \in (p_{in}, p_{rt})$ the inequality in Equation (32) holds. This completes the proof.