CS 153	**Current Topics In Theoretical Computer Science**	**Spring 2022**
	Problem Set 1	
Out: April 21		*Due:* **May 6**

Reminder: you are encouraged to work in small groups; however you must turn in your own write-up and note with whom you worked. You may consult any materials related to this course. The full honor code guidelines and collaboration policy can be found in the course syllabus.

Please attempt all problems. **Please turn in a hard copy or email me your solutions.**

1. Prove that $R(\langle 2,2,2\rangle) \geq 6$.

2. Suppose there are elements $a_{i,j'}$, $b_{j,k'}$, $c_{k,i'}$ of a group G, $i, i' \in [n], j, j' \in [m], k, k' \in [p]$, with the property that

$$a_{i,j'}b_{j,k'}c_{k,i'} = 1 \quad \Leftrightarrow \quad i = i', j = j', k = k'.$$

   Show that there exists sets $X, Y, Z \subseteq G$ with sizes n, m, p, respectively, that satisfy the triple product property; i.e., for $x, x' \in X, y, y' \in Y, z, z' \in Z$,

$$x^{-1}x'y^{-1}y'z^{-1}z' = 1 \quad \Leftrightarrow \quad x = x', y = y', z = z'.$$

3. Show that if $A \subseteq G$ is an abelian subgroup, and X, Y, Z satisfy the triple product property in G, then

$$|X||Y||Z| \leq |A| \cdot \left(\frac{|G|}{|A|}\right)^3.$$

   Hint: consider random translations of the sets X, Y, Z.