CS 153 Current topics in theoretical computer science		**Spring 2012**
	Problem Set 1	
Out: May 3		*Due:* **May 10**

Reminder: you are encouraged to work in small groups; however you must turn in your own write-up and note with whom you worked. The solutions to some of these problems can be found in various online course notes and research papers. Please do not search for or refer to these solutions.

1. Prove that $R(\langle 2,2,2\rangle) \geq 6$.

2. Let $\{G_i\}$ be a family of finite groups. Suppose that $X_i, Y_i, Z_i \subseteq G_i$ satisfy the triple product property, and that $\sup_i\{\log_{|G_i|}(|X_i| \cdot |Y_i| \cdot |Z_i|)\} = 3/2$. As an example, the construction in the symmetric group satisfies these conditions.

   Let D_i denote the maximum dimension of an irreducible representation of G_i. Recall that we always have $D_i < |G|^{1/2}$. Prove that if for all i, $D_i < |G_i|^{1/2-\epsilon}$ for any $\epsilon > 0$, then $\omega = 2$.

3. Recall that if one can find subsets $A_1, \ldots, A_n, B_1, \ldots, B_n$ of an abelian group G satisfying

   (a) for all i, $|A_iB_i| \geq |A_i||B_i|$, and
   (b) for all i and $j \neq k$, $A_iB_i \cap A_jB_k = \emptyset$,

   with $|G| \leq n^{2+o(1)}$ and $|A_iB_i| \geq n^{2-o(1)}$, then $\omega = 2$. Give such a construction in a *nonabelian* group G. This implies that there is no "group-theory" obstruction to this route to $\omega = 2$.

4. Recall that a strong USP is a table with n columns and N rows, populated with elements of $\{1,2,3\}$, which we may equivalently view as a family of N partitions (a_i, b_i, c_i) of $[n]$, satisfying the following property: if $\pi, \tau \in S_N$ are not both the identity, then there exists i for which some $j \in [n]$ is in *exactly two* of the sets $a_i, b_{\pi(i)}, c_{\tau(i)}$.

   In class we constructed strong USPs with $N \geq 4^{(1/3)(n-o(n))}$, which had only two symbols from $\{1,2,3\}$ occurring in each column. Prove that any strong USP with this property satisfies $N \leq 4^{n/3}$.

5. Say that a 0/1 tensor T has *pseudorank* r if the minimum rank tensor T' whose support equals the support of T, has rank r. It is a consequence of a theorem of Strassen that if the pseudorank of $\langle n,n,n\rangle = O(n^{2+\epsilon})$ for all $\epsilon > 0$, then $\omega = 2$.

   (a) Give an example of an $n \times n$ 0/1 matrix with rank $n-1$ but pseudorank 2. Hint: in one example, every matrix entry is a root of unity.
   (b) Consider the 0/1 tensor $T \in \mathbf{R}^{2\times 2\times 2}$ with slices:

   $$a_1 = \begin{array}{|c|c|}\hline 1 & 1 \\ \hline 0 & 0 \\ \hline\end{array} \quad a_2 = \begin{array}{|c|c|}\hline 0 & 1 \\ \hline 1 & 1 \\ \hline\end{array}$$

   Prove that $R(T) = 3$ but the pseudorank of $T \leq 2$.

Is the pseudorank of $\langle 2, 2, 2 \rangle < 7$? This question is not part of the problem set but I would like to know the answer!