

Course Summary and Syllabus

Lecturer: Chris Umans

Date: March 30

Course summary:

This year's theme will be "hardness of approximation."

In the first part of the course, we will come to understand the connection between provable (under a complexity assumption) limits on the quality of approximation algorithms, and *probabilistically checkable proofs* (PCPs). We will then prove the PCP Theorem in its entirety, drawing on a presentation that has emerged in recent works that is simpler and more modular than the original proof. We will end with Håstad's celebrated 3-bit PCP. Time permitting, we may cover Dinur's self-contained proof via "gap amplification" as well.

In the second part of the course, we will consider *unique games*, a particular strengthening of PCPs, which is conjectured to be NP-hard. We will prove the astonishing result that the Goemans-Williamson 0.878...-approximation algorithm for MAX CUT is optimal under the Unique Games Conjecture. We will also study a recent result that shows that for a broad class of *constraint satisfaction problems*, a simple SDP-based approximation algorithm is best-possible, again under the Unique Games Conjecture. We will conclude with recent results giving approximation algorithms for unique games that (perhaps) suggest a route to disproving the conjecture.

Course Information:

- Instructor: Chris Umans (umans@cs.caltech.edu)
- Lectures: Tuesdays and Thursdays 1:00 – 2:25 in Annenberg 314
- Office hours: TBD
- Text: None. A significant portion of the course will follow course notes from a DIMACS tutorial, available here <http://arxiv.org/abs/1002.3864>. The webpage will be updated regularly with links to relevant papers.
- Webpage: <http://www.cs.caltech.edu/~umans/cs153/>

Prerequisite: This course is pitched at a beginning graduate level, but both undergrads and grad students are encouraged to attend. Prerequisites are mathematical maturity and curiosity. The course is intended to be largely self-contained, but exposure to elementary probability and algebra, as well as material covered in CS21, CS38 and CS151, is helpful.

Course requirements:

Course participants should attend/participate in lectures (20%), complete 2-3 short problem sets (20%), and read and present a relevant research paper at the end of the term (60%).

(Very) tentative lecture schedule:

Lecture	Date	Topic
1	Mar 30	Introduction; approximation algorithms
2	Apr 1	Hardness of approximation and PCPs; label cover
3	Apr 6	Proof of the PCP theorem
4	Apr 8	Proof of the PCP theorem
5	Apr 13	Proof of the PCP theorem
6	Apr 15	Håstad's 3-bit PCP and hardness of MAX-3-SAT, MAX-3-LIN
7	Apr 20	Håstad's 3-bit PCP and hardness of MAX-3-SAT, MAX-3-LIN
8	Apr 22	Unique Games and semidefinite programming, the Unique Games Conjecture (UGC)
9	Apr 27	no class: out of town
10	Apr 29	UGC-hardness for MAX-CUT
11	May 4	UGC-hardness for MAX-CUT
12	May 6	UGC-hardness and SDP-approximations for CSPs
13	May 11	UGC-hardness and SDP-approximations for CSPs
14	May 13	UGC-hardness and SDP-approximations for CSPs
15	May 18	no class?: out of town?
16	May 20	no class?: out of town?
17	May 25	algorithms for Unique Games
18	May 27	algorithms for Unique Games
19	Jun 2	paper presentations
20	Jun 4	paper presentations

Possible topics/papers for presentation will be listed on the webpage.