CS284b Assignment #1 **Due:** Friday, January 19

The purpose of this assignment is to familiarize you with Java and some of the mathematical tools that have been covered in class. You may use another programming language if you wish (such as C or C++), but the sample code that we will supply will be in Java. Consult the class web page for additional information on Java.

Programming

1. Modify the *DrawSmooth* class to draw a Bezier curve of order n through $n+1$ user-supplied points. Draw the curve by performing binary subdivisions, not necessarily at the midpoints, to a depth specified by the user. Approximate each of the resulting curves by a stright line. Alow for interactive manipulation of the initial control points, as in the original program.
2. Compute individual points along the curve corresponding to specified parameters in the range $[0,1]$. Allow the parameter to be specified interactively (perhaps with a slider), and indicate the corresponding point on the curve graphically.

Theory

1. Let $p_1, p_2, \ldots, p_k$ be points in R^n. Prove (a) that convex combinations of the points remain within the convex hull of the points, and (b) that the collection of all affine combinations of the points defines an affine subspace of R^n.
2. Consider the interval $\Delta := [s,t]$ and define the generalized Bernstein-Bézier polynomials $B_i^{\Delta,n}(u) = \begin{pmatrix} n \\ i \end{pmatrix} \left(\frac{u-s}{t-s}\right)^i \left(1 - \frac{u-s}{t-s}\right)^{n-i}$ and prove the following facts:
   - $1 = \sum_{i=0}^n B_i^{\Delta,n}(u)$
   - $B_i^{\Delta,n}(u) \geq 0$ for $u \in [s,t]$
   - $B_i^{\Delta,n}(u) = \frac{u-s}{t-s} B_{i-1}^{\Delta,n-1}(u) + \left(1 - \frac{u-s}{t-s}\right) B_i^{\Delta,n-1}(u)$
   - $B_i^{\Delta,n}(u) = B_{n-i}^{\Delta,n}(t+s-u)$
   - $B_i^{\Delta,n}(u)$ has a local maximum at $\frac{i}{n}(t-s)+s$
3. For the quartic planar curve $G(u) := (6u^2, 7u^4 + 2u^3 - 8u^2 - 4u + 1)$
   a compute its polarform;
   b find it's Bézier points with respect to $[2,4]$;
   c use the de Casteljau algorithm to compute the value and derivative of G at $5/2$. Write down the entire de Casteljau pyramid.