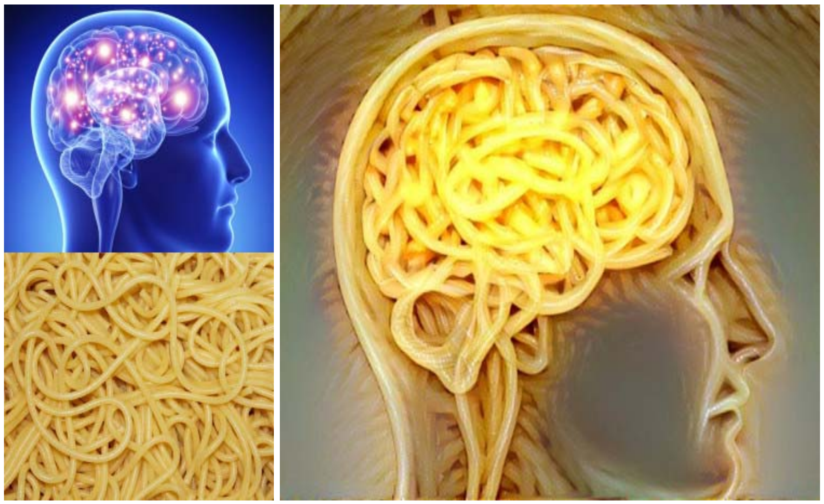
Deep Learning

Impressive results

https://deepart.io/
https://deepdreamgenerator.com/




A Neural Algorithm of Artistic Style, Gatys et al, 2015

BUT **It is “alchemy”**

- We don’t know why algorithms work or why they don’t (no theory)
- Algorithms are developed through trial and error
- Some results are hard to replicate (many hyperparameters)
- Finding good architectures relies on guesswork
- Very deep networks (more 40 layers) are difficult to train with backpropagation
- Algorithms are not robust to adversarial examples

AI researchers allege that machine learning is alchemy

By Matthew Hutson | May. 3, 2018 , 11:15 AM

Ali Rahimi, a researcher in artificial intelligence (AI) at Google in San Francisco, California, took a swipe at his field last December—and received a 40-second ovation for it. Speaking at an AI conference, Rahimi charged that machine learning algorithms, in which computers learn through trial and error, have become a form of "alchemy." Researchers, he said, do not know why some algorithms work and others don't, nor do they have rigorous criteria for choosing one AI architecture over another. Now, in a paper presented on 30 April at the International Conference on Learning Representations in Vancouver, Canada, Rahimi and his collaborators document examples of what they see as the alchemy problem and offer prescriptions for bolstering AI's rigor.

"There's an anguish in the field," Rahimi says. "Many of us feel like we're operating on an alien technology."

Science Mag, May 2018

"Machine learning has become alchemy”
Ali Rahimi
NIPS 2017 Test of Time Award

Questions

Can the interface between NA and Game theory offer some insights?

Is there an approach that

- **Is amenable to some degree of analysis?**
- **Produces a network without guesswork?**
  (plug and play, no tweaking of hyperparameters, no guessing of the architecture)
- **Enables the training of very deep networks?**
  (50,000 layers or more) and the exploration of their properties
- **Provides some insight on developing a rigorous theory for deep learning?**

Initial results

Interface between Game Theory and NA $\Rightarrow$ **Deep Learning**

Gene Ryan Yoo

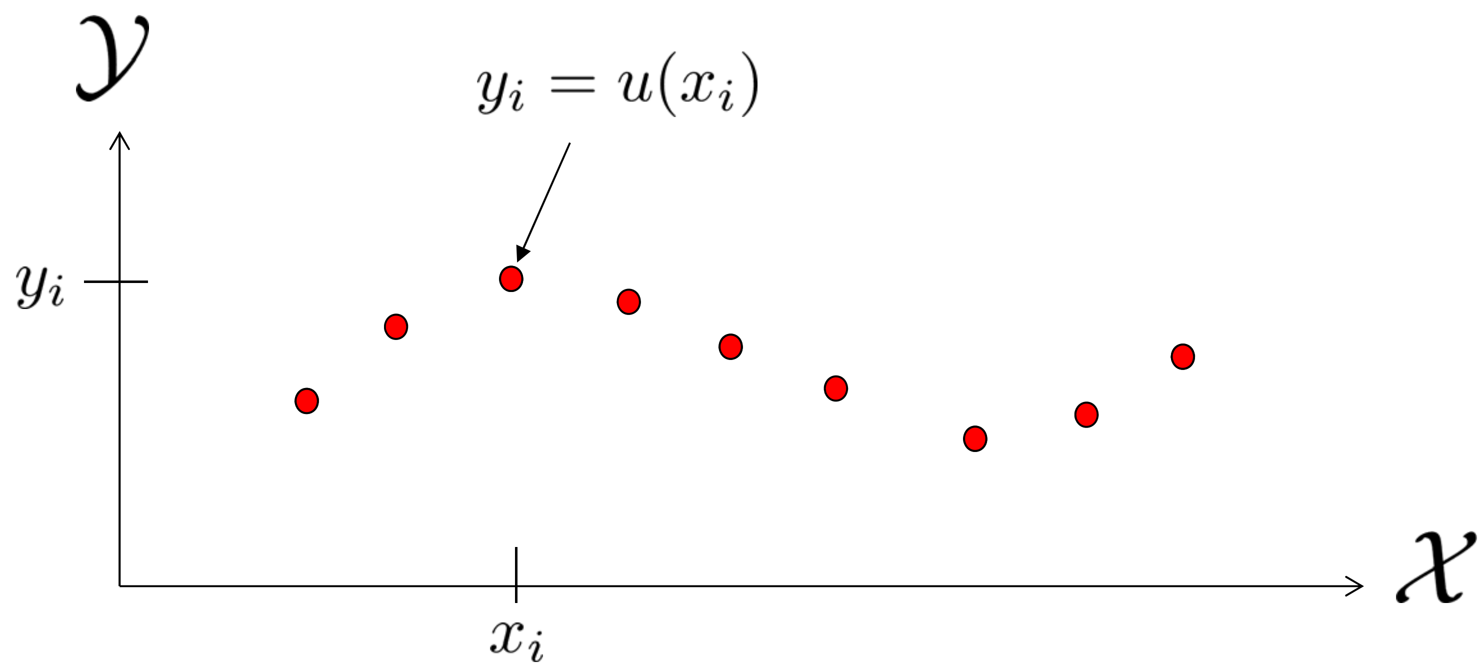
- Kernel Flows: from learning kernels from data into the abyss. H. Owhadi and G. R. Yoo, arXiv:1808.04475, 2018.

Learning is solving an interpolation problem

$$\mathcal{X} \xrightarrow{\quad u \quad} \mathcal{Y}$$

u : Unknown

Given $y_i = u(x_i)$ for $i = 1, \ldots, N$, approximate u

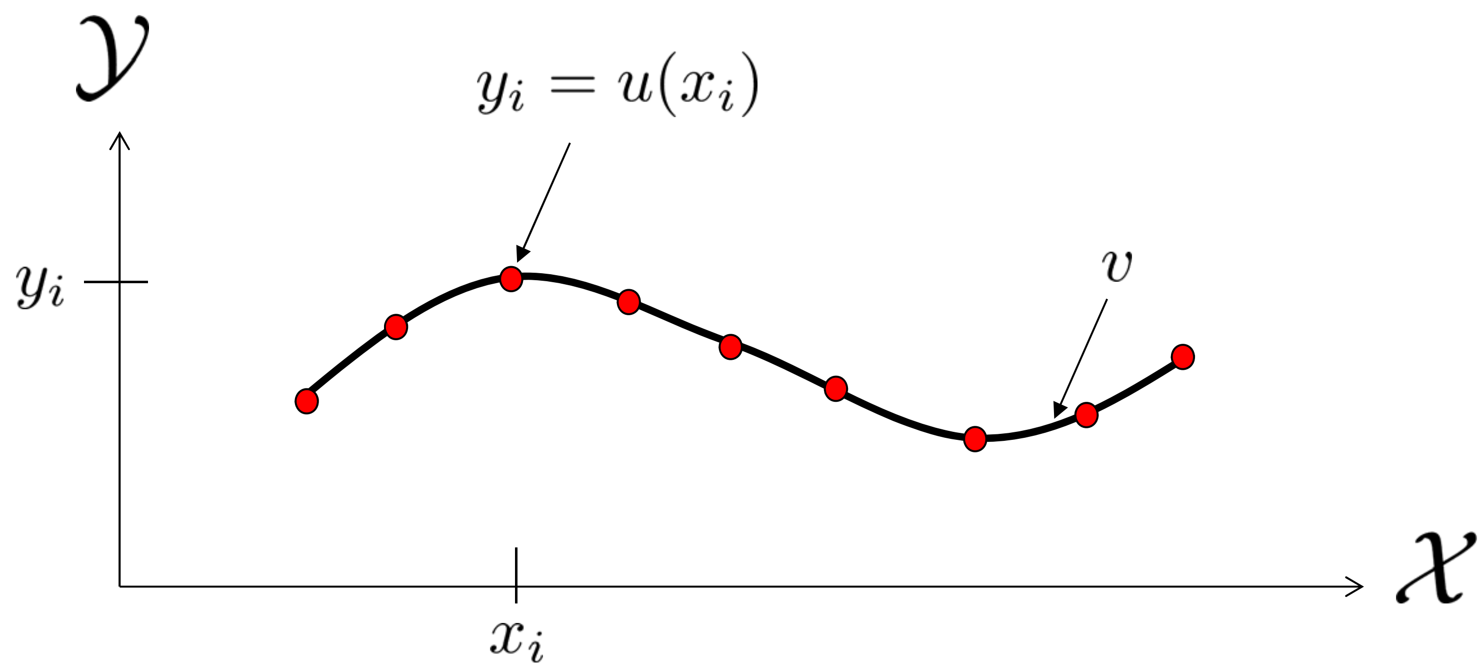


Solution: Kriging/GPR/SVM

Given kernel K approximate $u(x)$ with

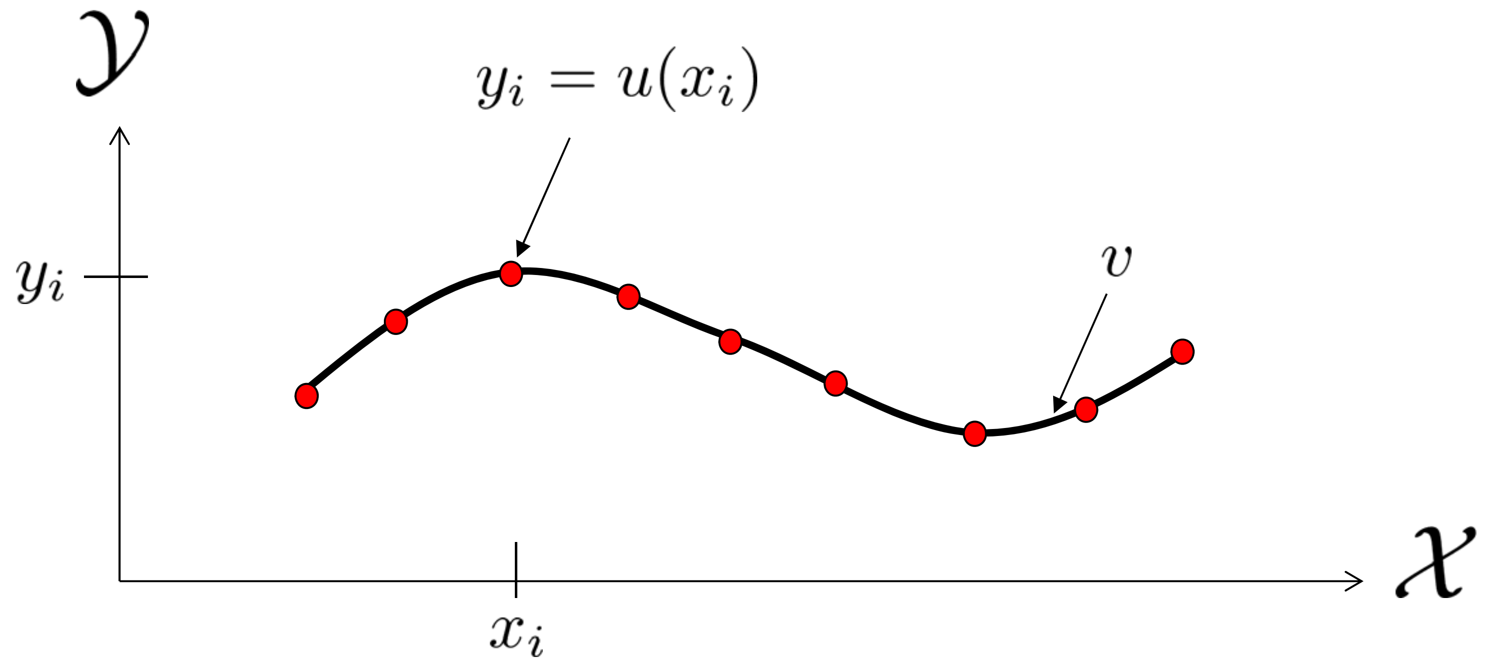
$$v(x) = \sum_i c_i K(x_i, x)$$

c such that $v(x_i) = y_i$ for all i



BUT

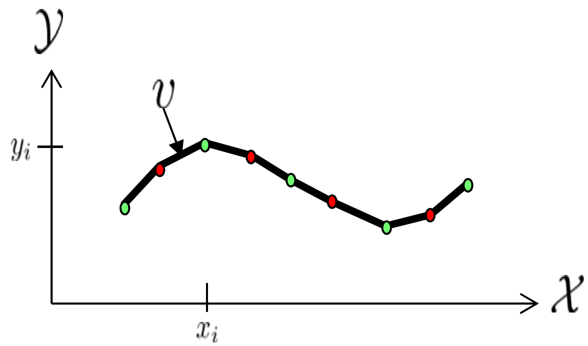
- **What if N is large?**
- **Which kernel do we pick?**



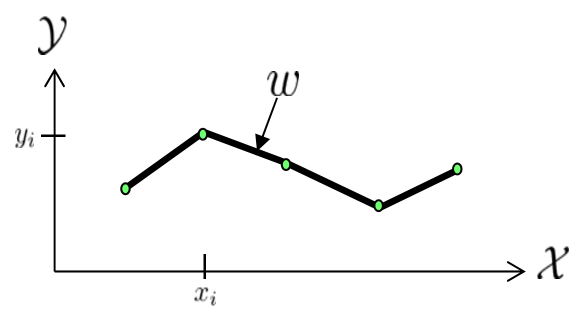
Premise

A kernel K is good if the number of interpolation points can be halved without significant loss in accuracy

v: Interpolate with K and N points



w: Interpolate with K and $N/2$ points



$$\boxed{\rho = \frac{\|v-w\|^2}{\|v\|^2}}$$

$$\|v\|^2 = \sup_\phi \frac{\left(\int \phi(x)v(x)\,dx\right)^2}{\int \phi(x)K(x,x')\phi(x')\,dx\,dx'}$$

Good kernel $\Longleftrightarrow$ Small ρ

Kernel Flow

Learns kernels of the form

$$\boxed{K_n(x,x') = K_1(F_n(x), F_n(x'))}$$

K_1: kernel (e.g. $K_1(x,x') = e^{-\frac{|x-x'|^2}{\gamma^2}}$)

F_n : Flow in input space

$$F_n : \mathcal{X} \to \mathcal{X}$$

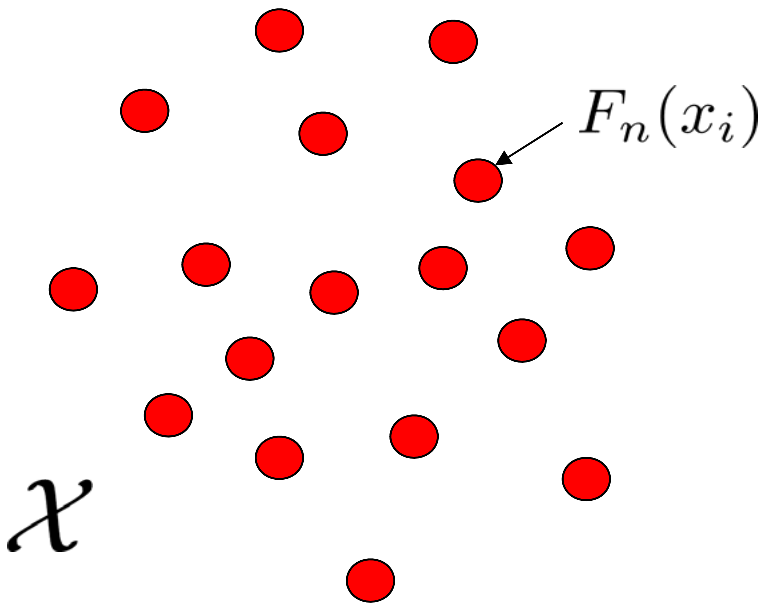
$$F_1 = I_d$$

$$F_n \xrightarrow[\uparrow \text{ Data}]{} F_{n+1}$$

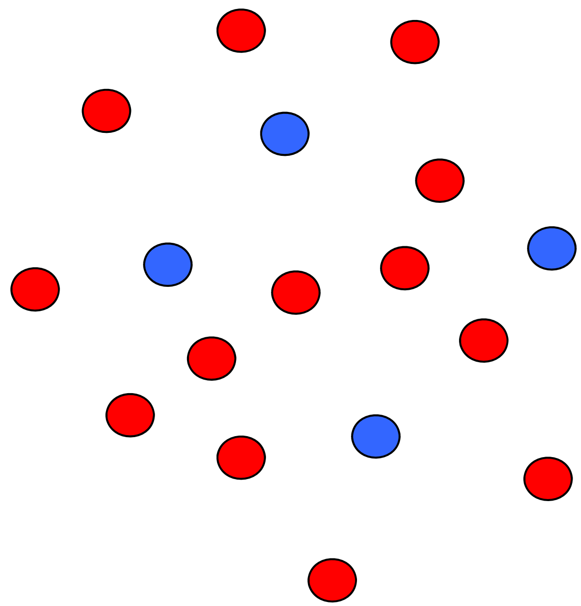
$\boxed{\text{Step } n \to n+1}$

$\boxed{\text{Assume } F_n \text{ known}}$

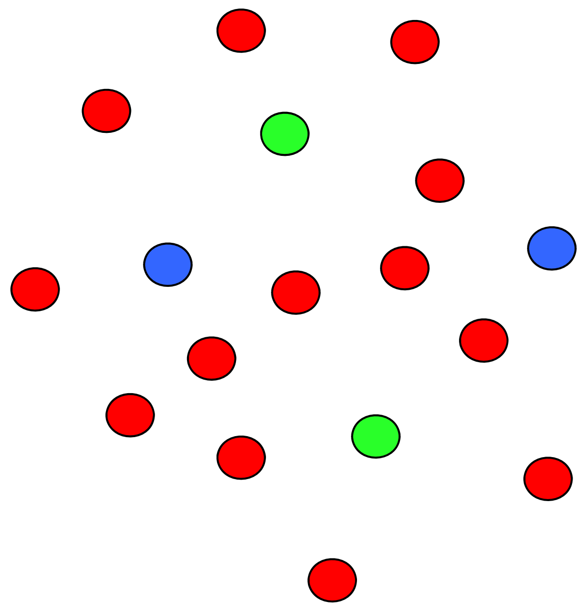
Images of the N training points under F_n



Select N_f at random out of N



Select $N_f/2$ at random out of N_f

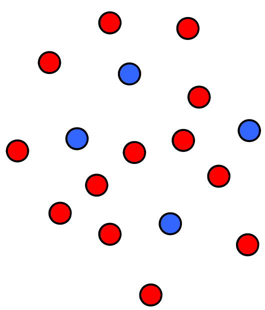


Player I

Selects the values/labels of the blue points $F_n(x_i)$ to be y_i (training labels)

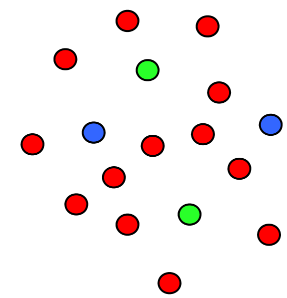
Player II

Sees values/labels y_i of the $N_c = N_f/2$ green points must predict the values of the blue points



Max

Min

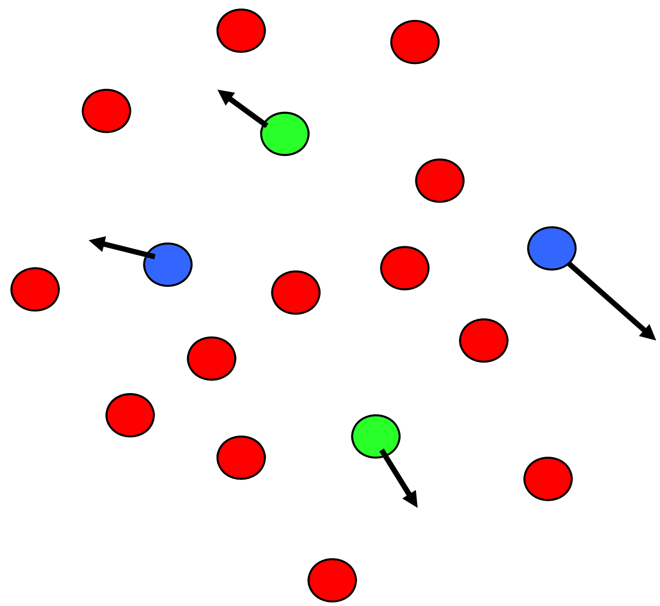


$$\rho$$

ρ: Relative error in $\|\cdot\|$ norm

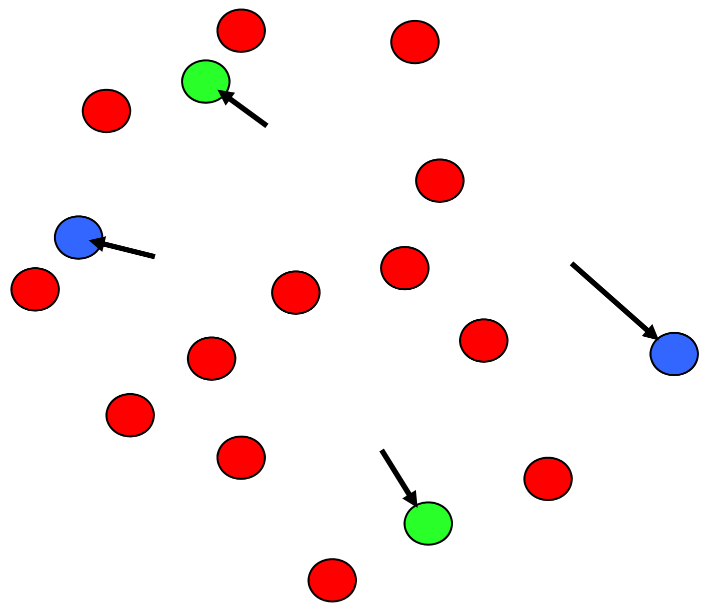
$\|\cdot\|$: RKHS norm associated with K_1

Move the N_f points in the gradient descent direction of ρ

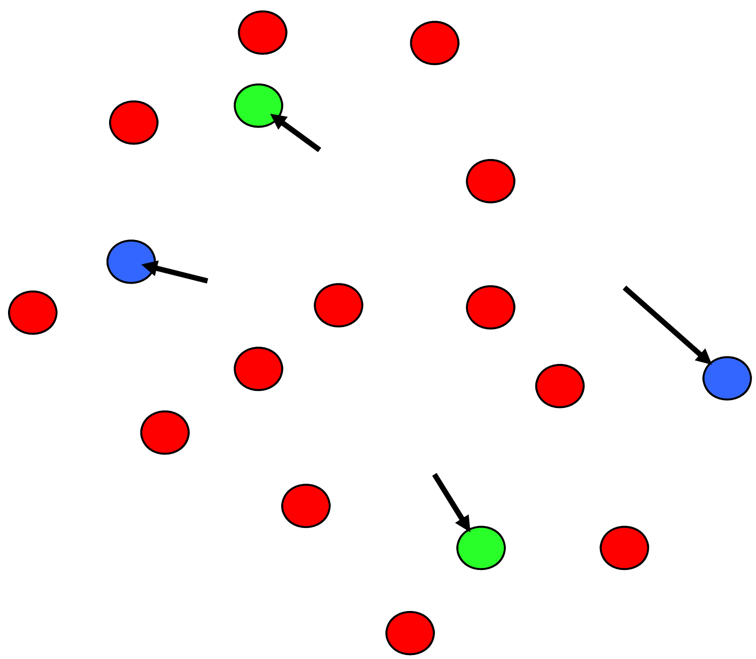


Rig the game in favor of Player II

Move the N_f points in the gradient descent direction of ρ



Move the remaining $N - N_f$ points
via interpolation with kernel K_1

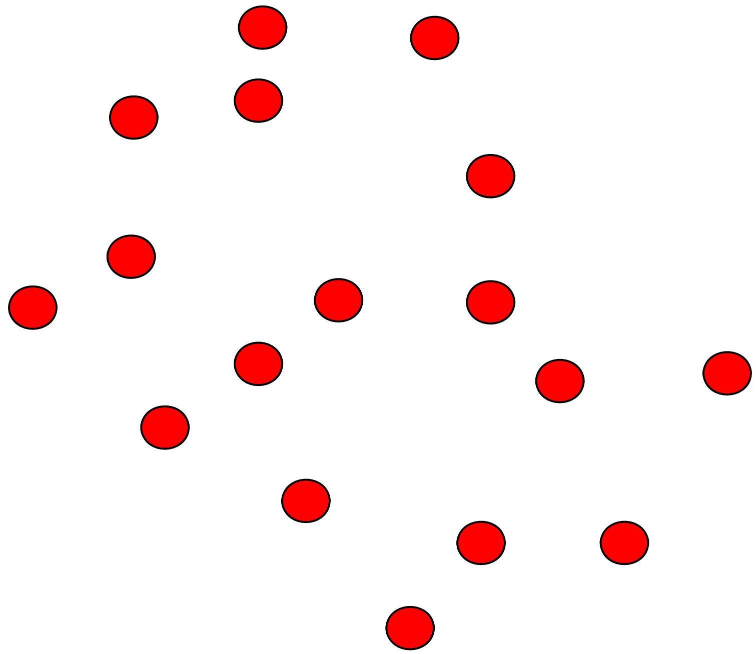


Move any point x
via interpolation with kernel K_1 $\Rightarrow$ F_{n+1}

Repeat

F_{n+1} known

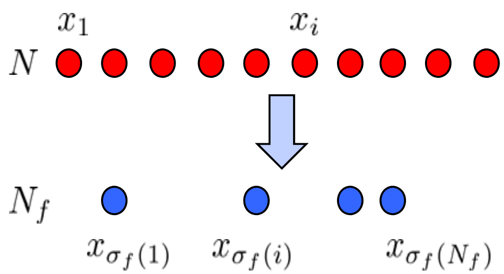
Images of the N training points under F_{n+1}

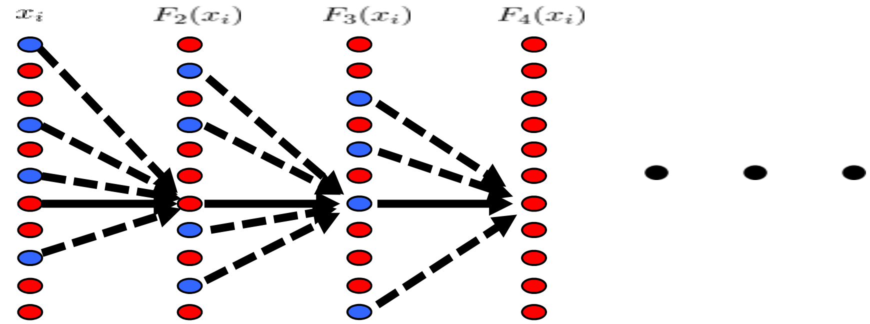


Kernel Flow

Produces a deep hierarchical kernel of the form

$$\boxed{K_n(x,x') = K_{n-1}(x+\epsilon \sum_{i=1}^{N_f} c_i K_{n-1}(x_{\sigma_f(i)},x), x'+\epsilon \sum_{i=1}^{N_f} c_i K_{n-1}(x_{\sigma_f(i)},x'))}$$





Randomized bottomless network

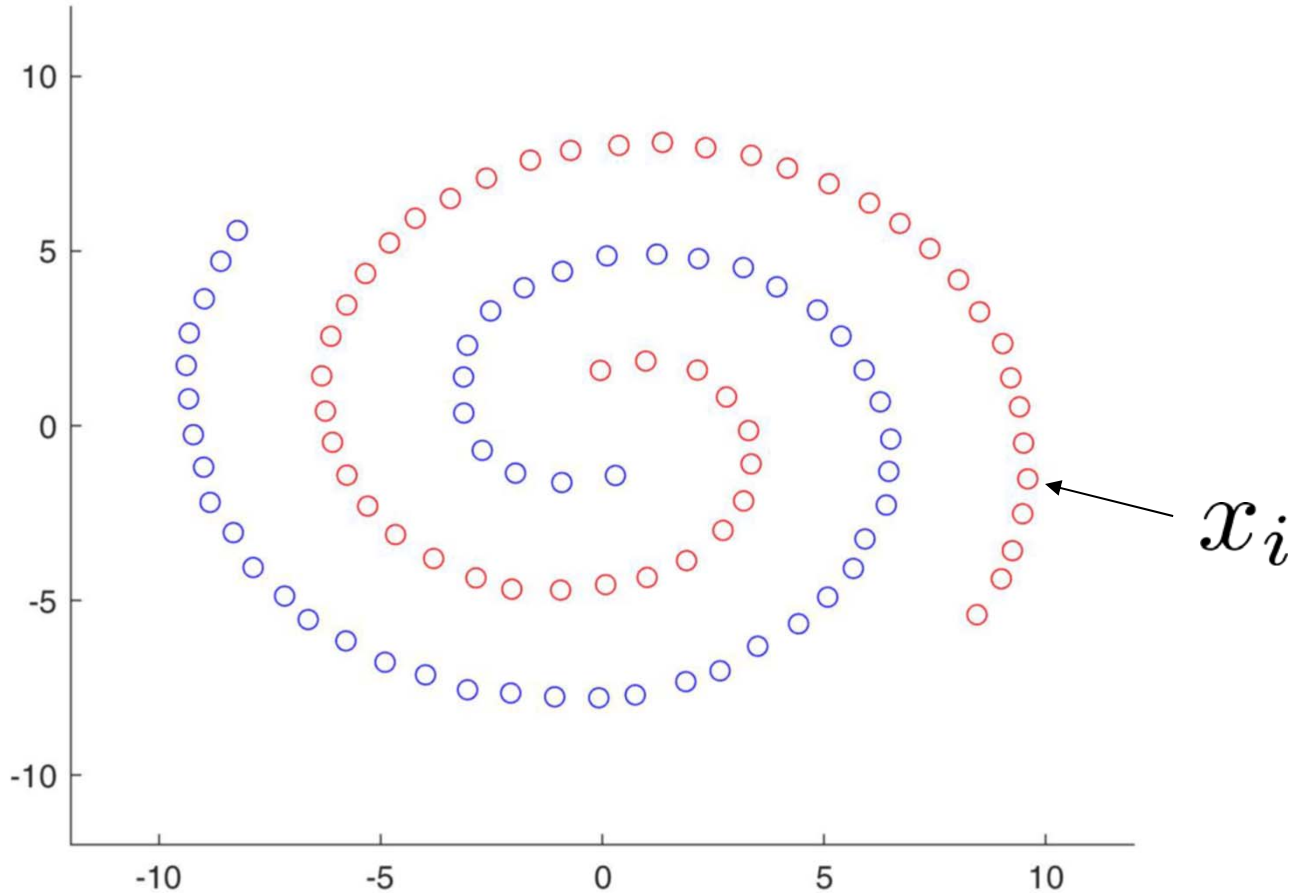
and a flow of the form

$$\boxed{F_{n+1} = (I_d + \epsilon G_{n+1}) \circ F_n}$$

$$\boxed{G_{n+1}(x) = \sum_{i=1}^{N_f} c_i K_1(F_n(x_{\sigma_f(i)}), x)}$$

Identified as the steepest gradient descent direction of ρ.

Application: Swiss Roll Cheesecake



$N = 100$ data points $x_i \in \mathbb{R}^2$

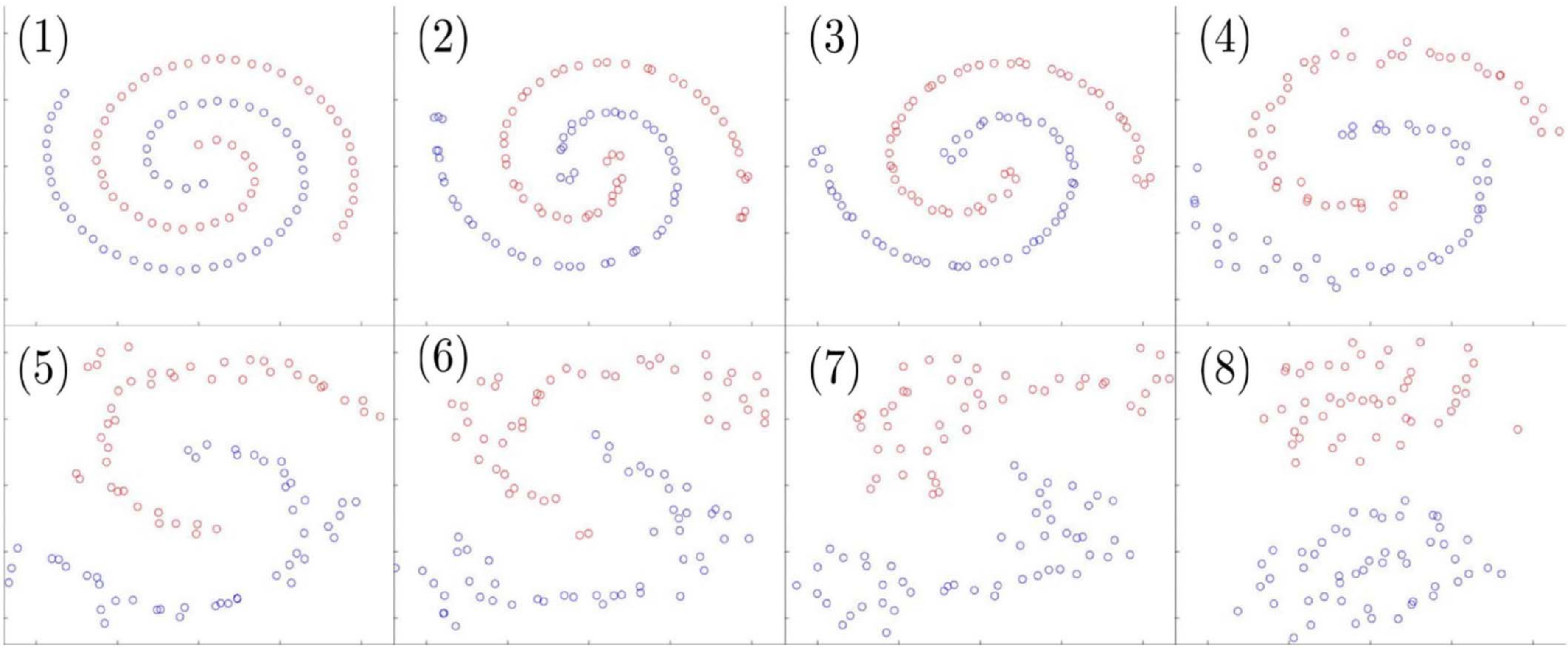
$y_i = -1$ if point at x_i is red

$y_i = +1$ if point at x_i is blue

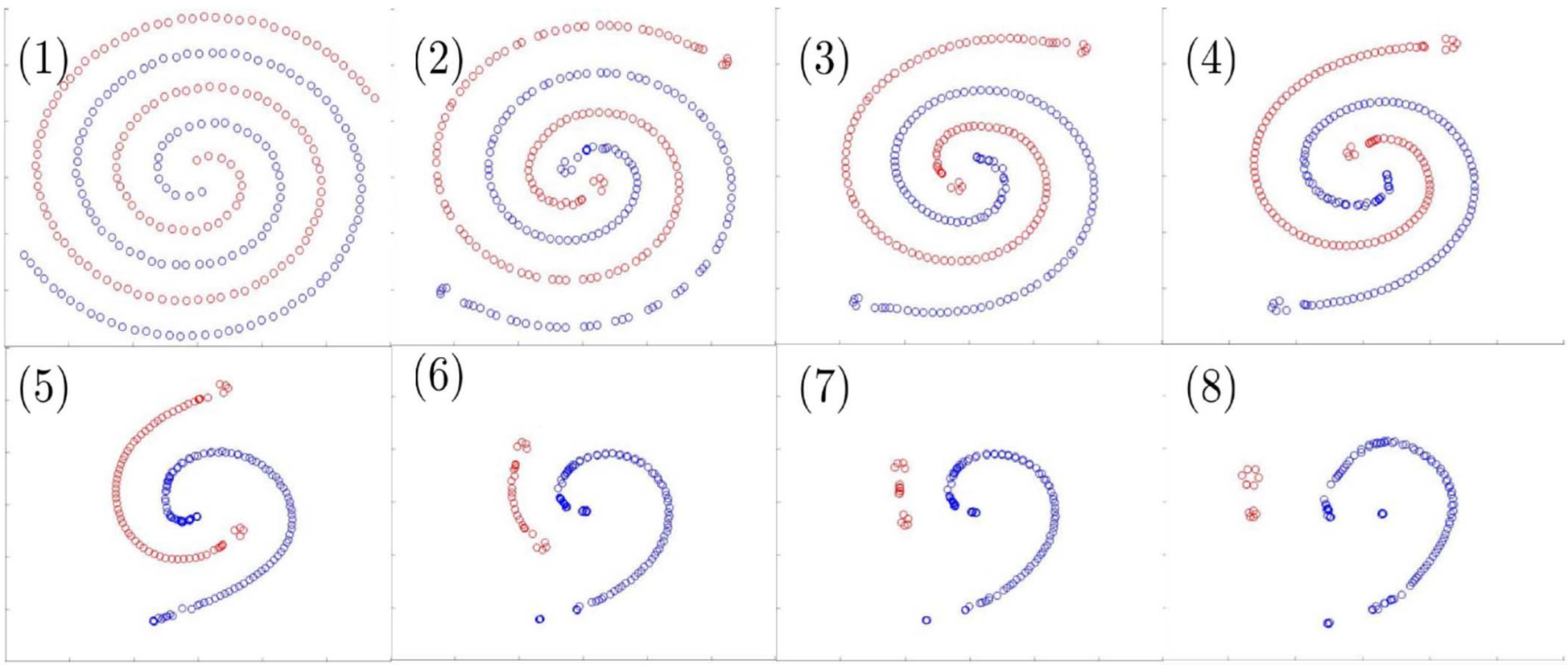
Objective:

Visualize $n \to F_n(x_i)$

$F_n(x_i)$ Gaussian Kernel, $N_f = N$



$F_n(x_i)$ Gaussian Kernel, $N_f = N$, large ϵ



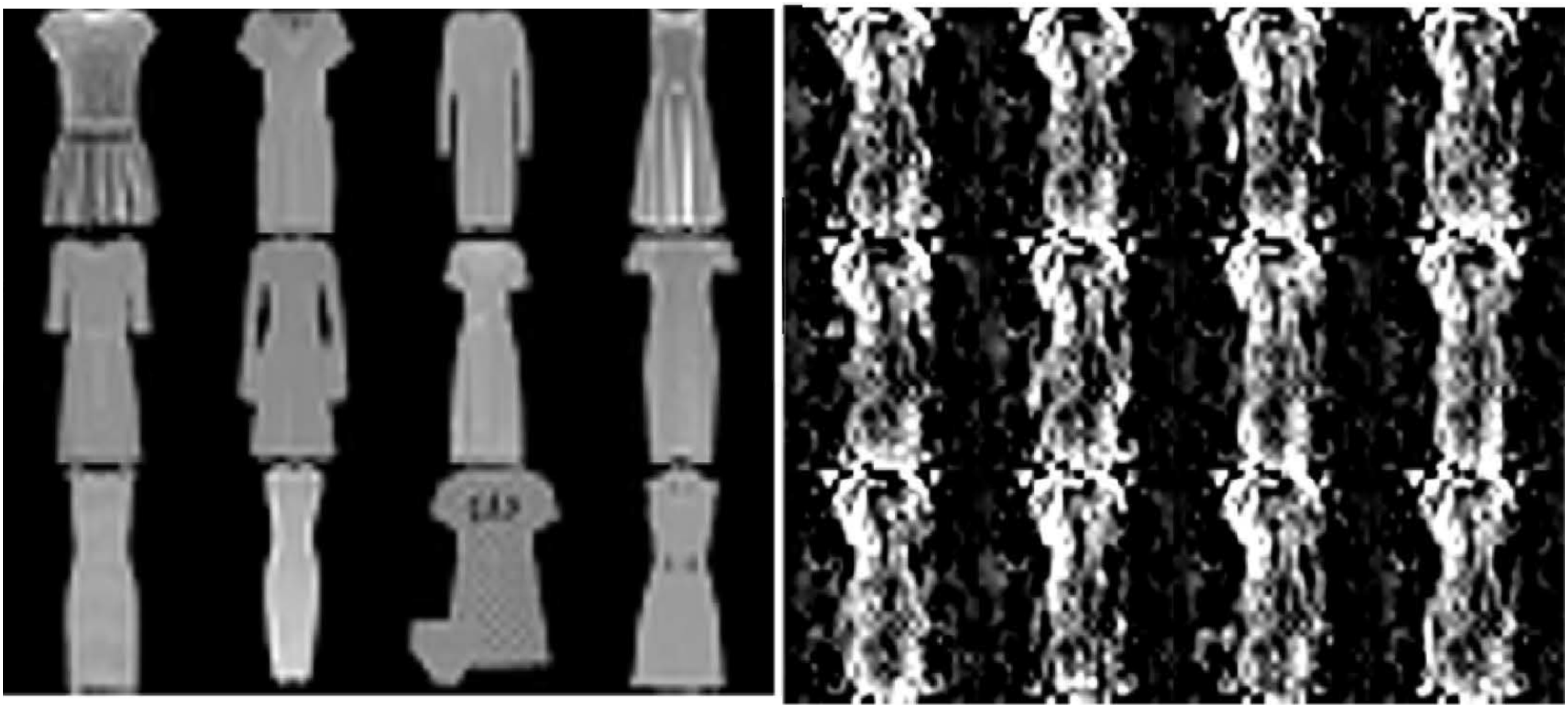
Application to Fashion-MNIST

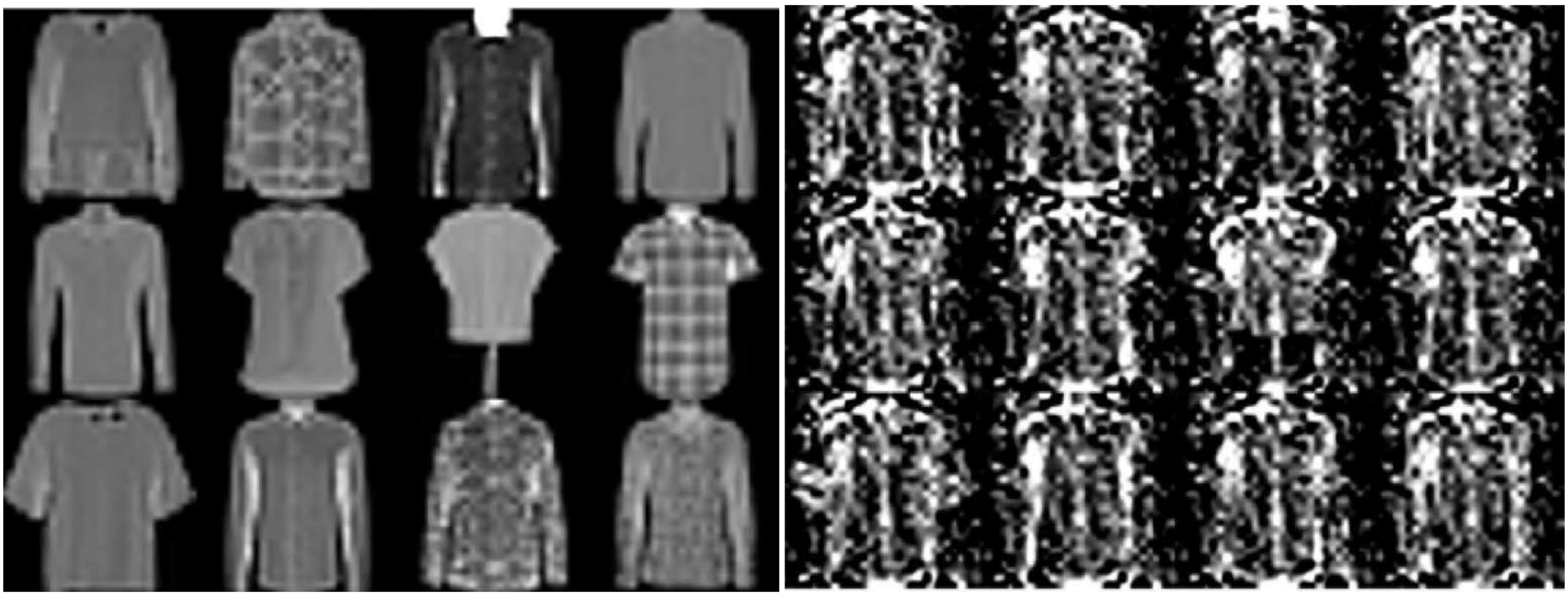
$$N = 60000$$

$$N_f = 600$$

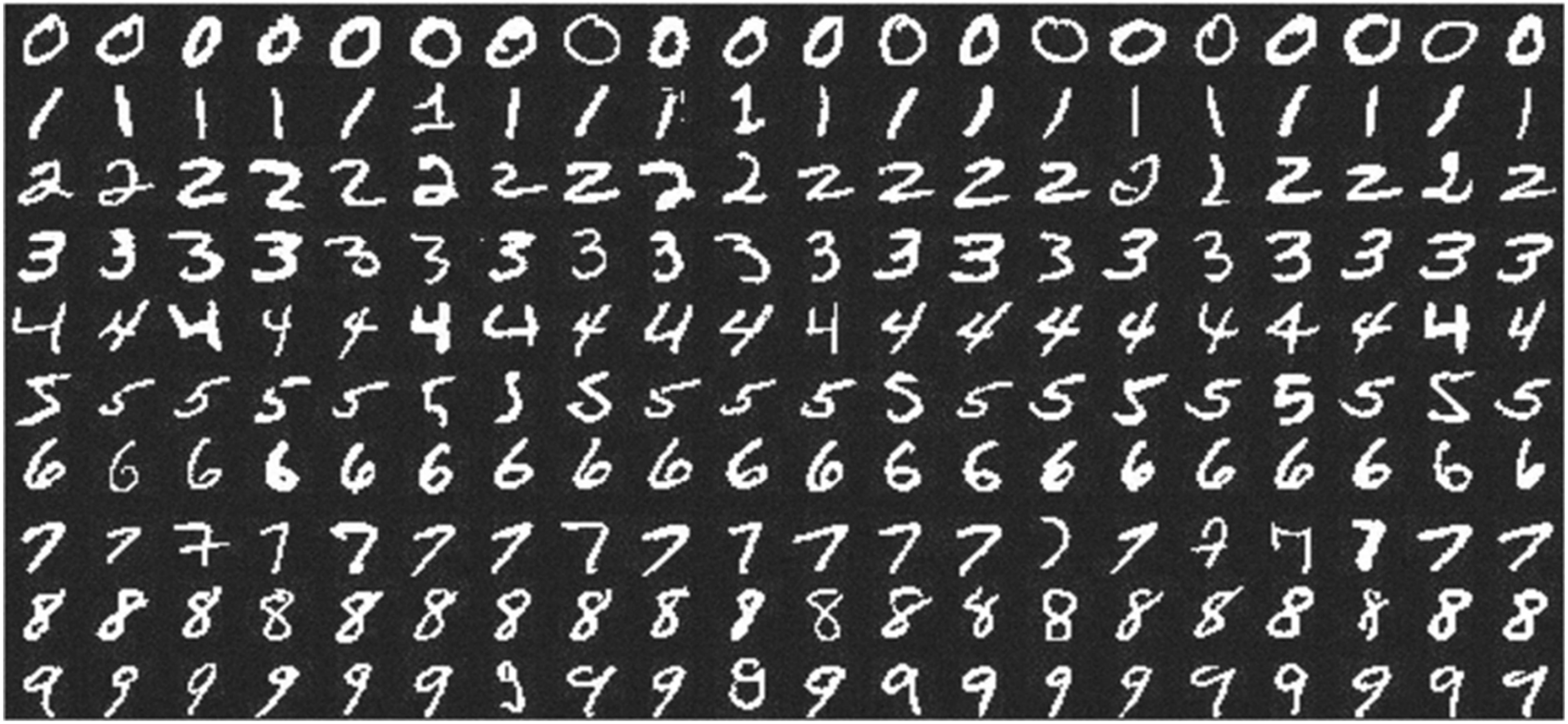
12000 layers, large steps

50000 layers, small steps





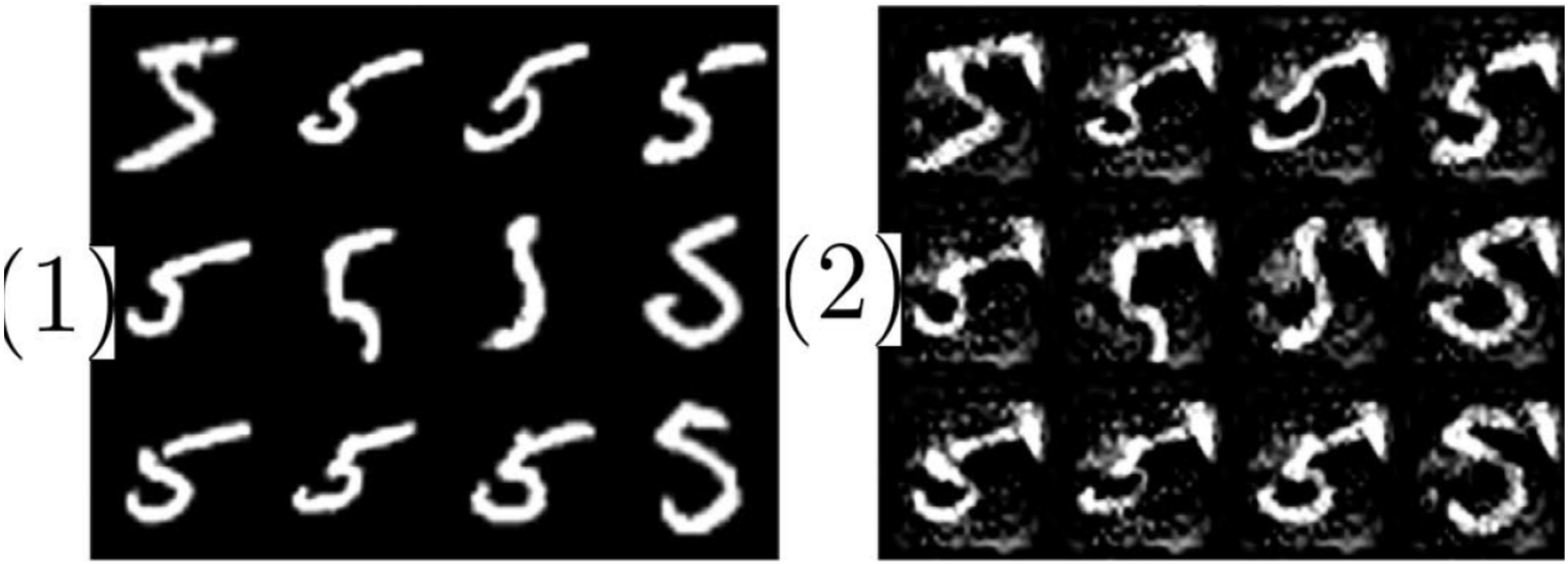
Application to MNIST



$N = 60000$

$N_f = 600$

12000 layers


(1)
(2)

(3)

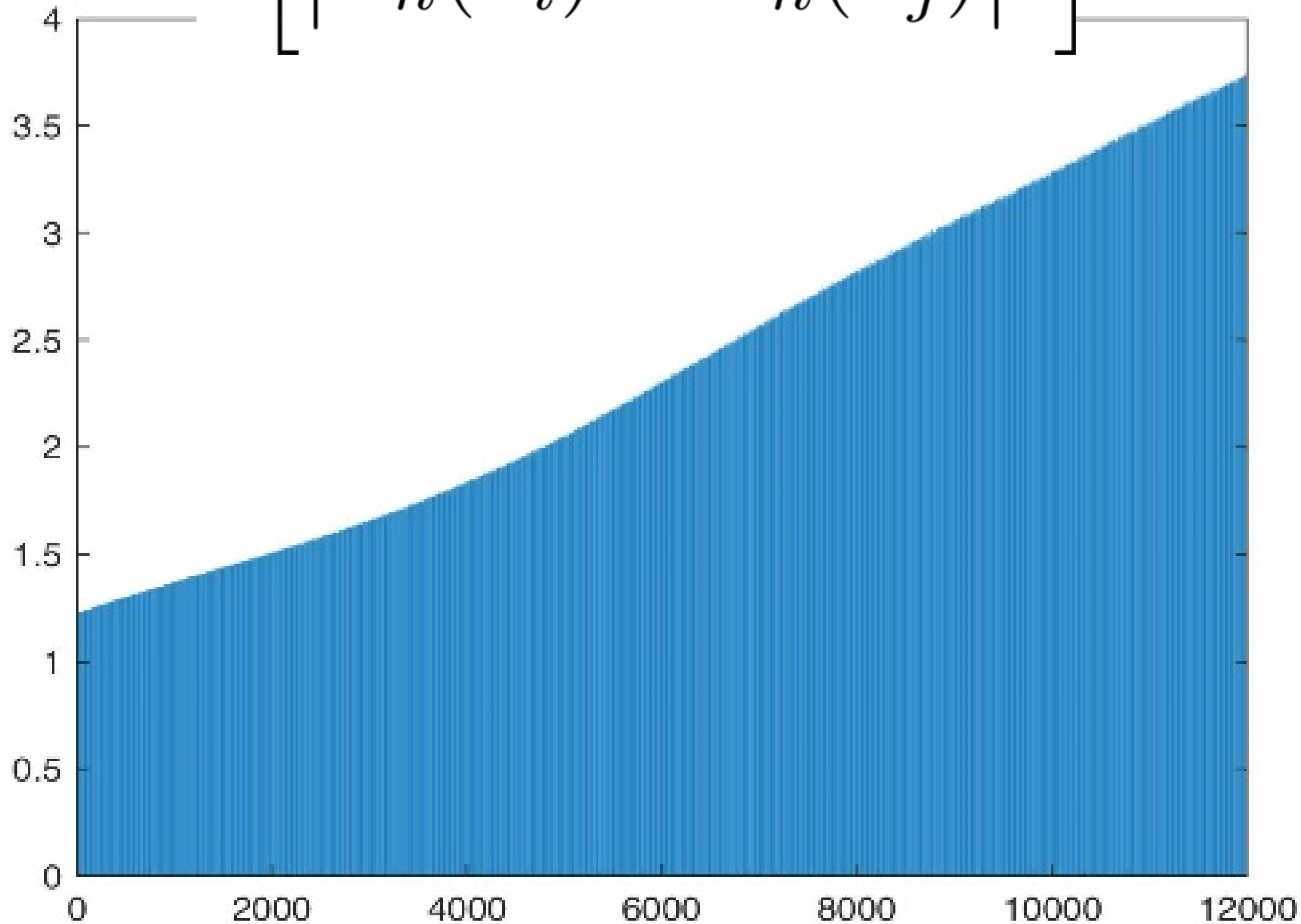
(4)


MNIST

Average distance, inter-class

$$y_i \neq y_j$$

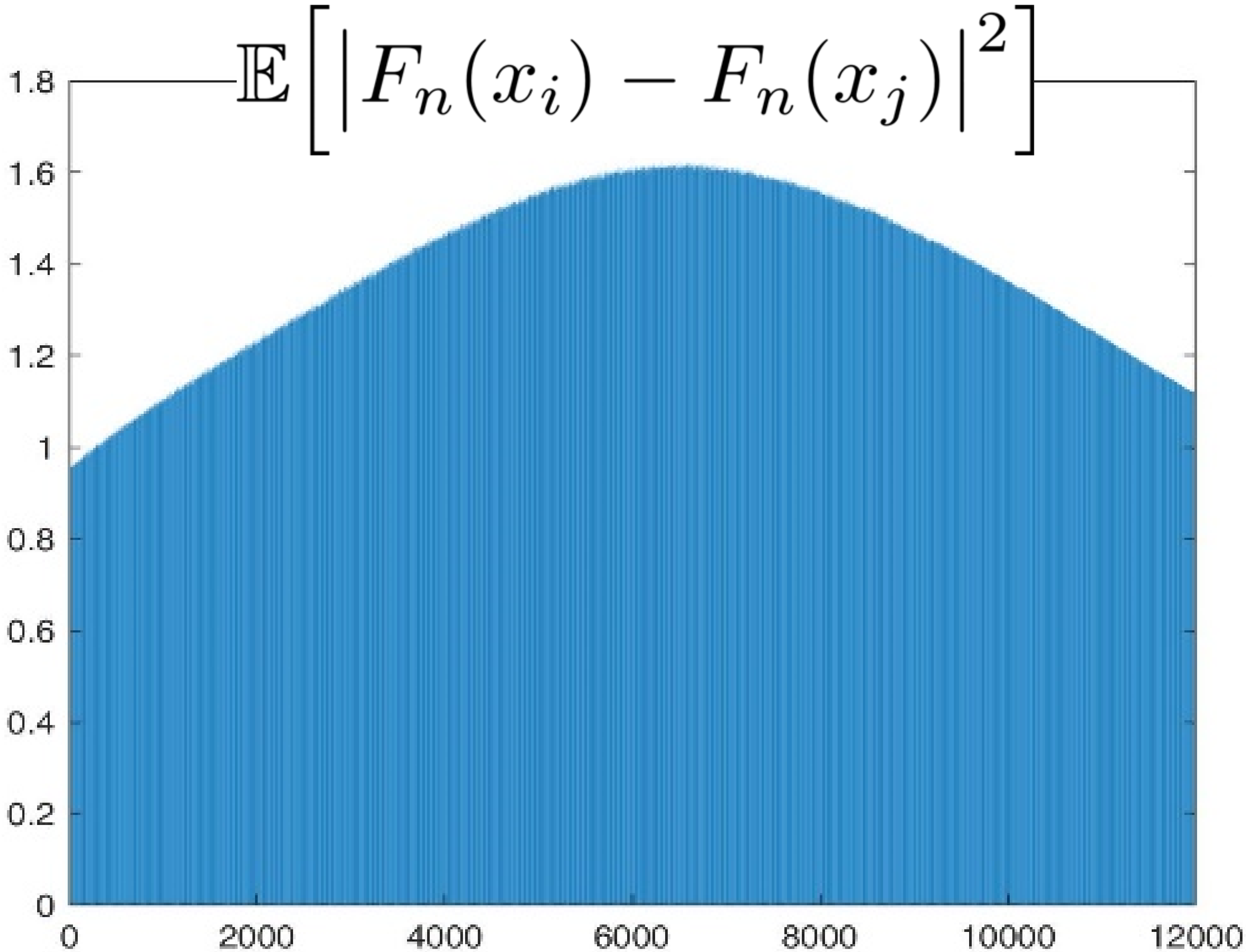
$$\mathbb{E}\Big[\big|F_n(x_i) - F_n(x_j)\big|^2\Big]$$



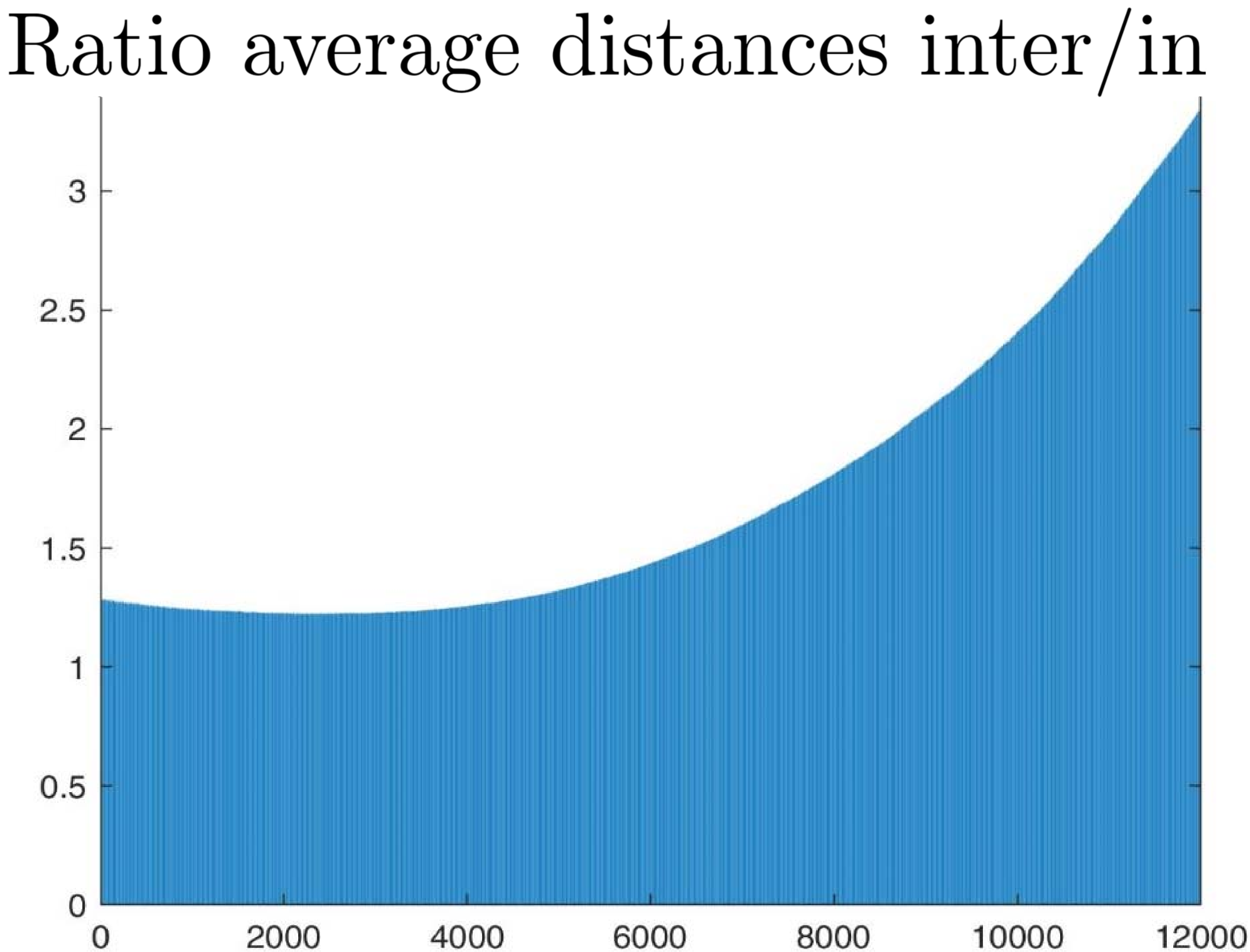
MNIST

Average distance, in-class

$y_i = y_j$

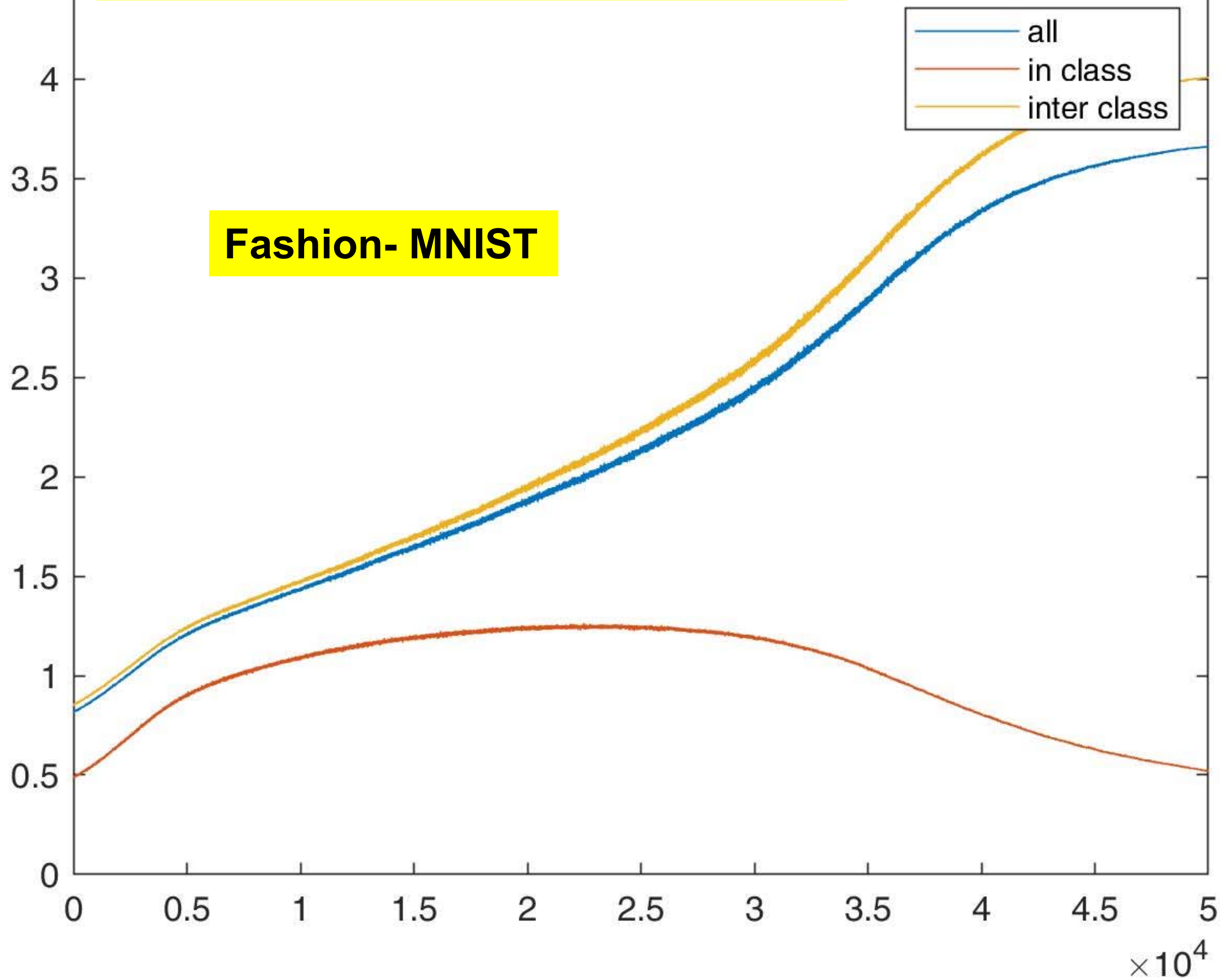


MNIST



Ratio average distances inter/in
3
2.5
2
1.5
1
0.5
0
0
2000
4000
6000
8000
10000
12000

Average distance vs n



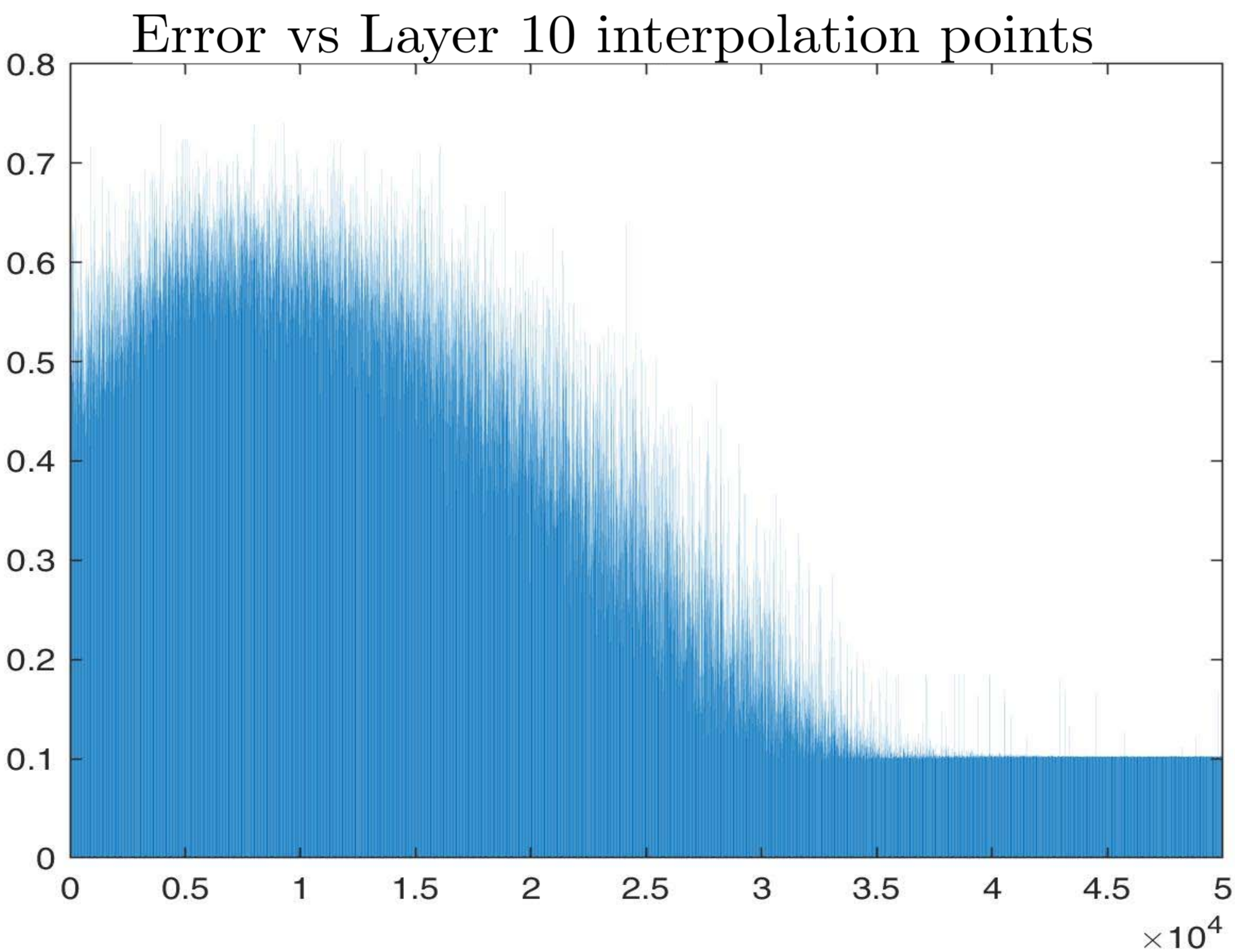
Classify 10000 test points

Use kernel K_n
and N_I interpolation points
selected at random

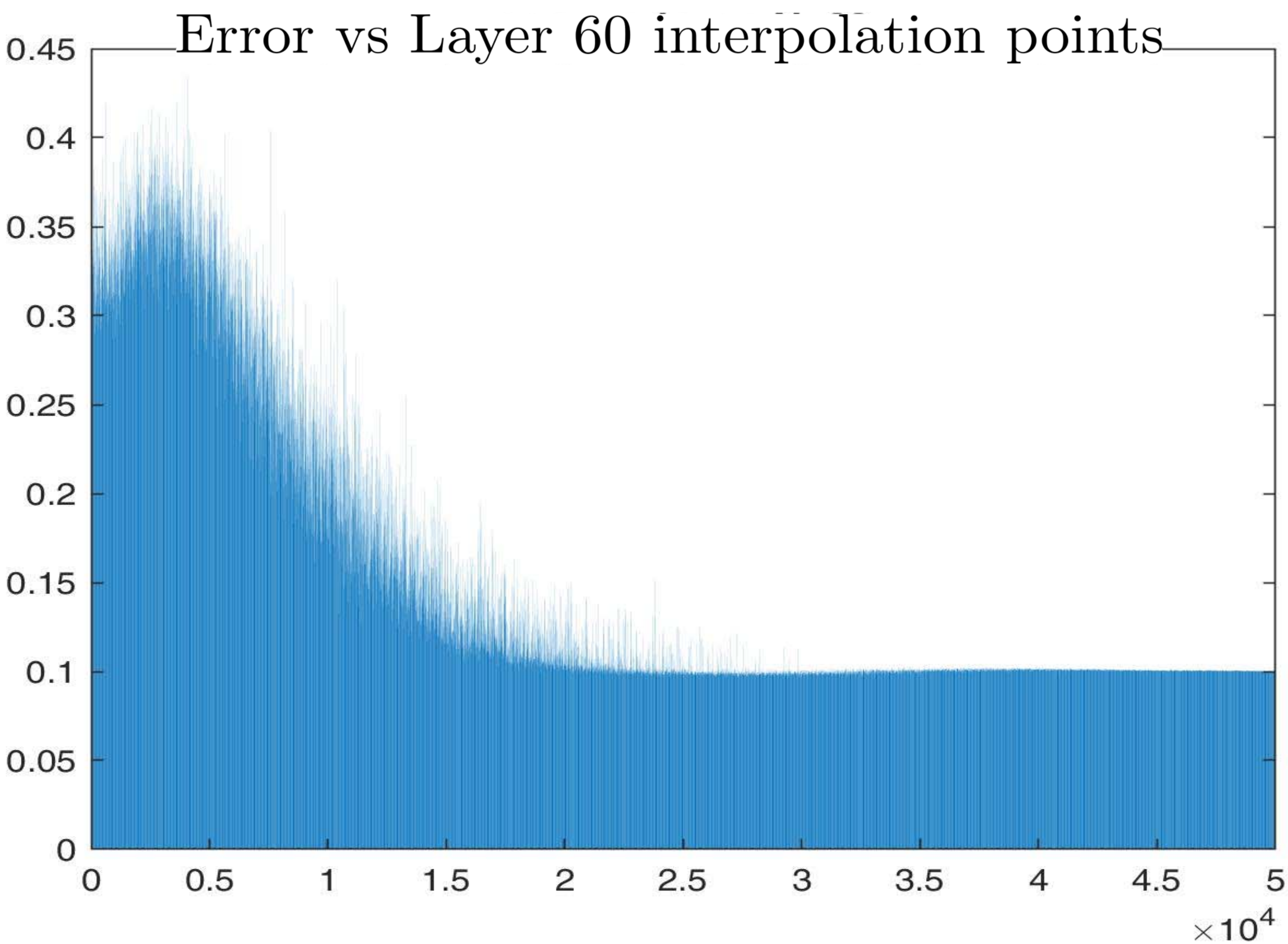
$$N_I = 6000, 600, 60, 10$$

$N_I = 10$ ⟺ Interpolate with only 1 point per class

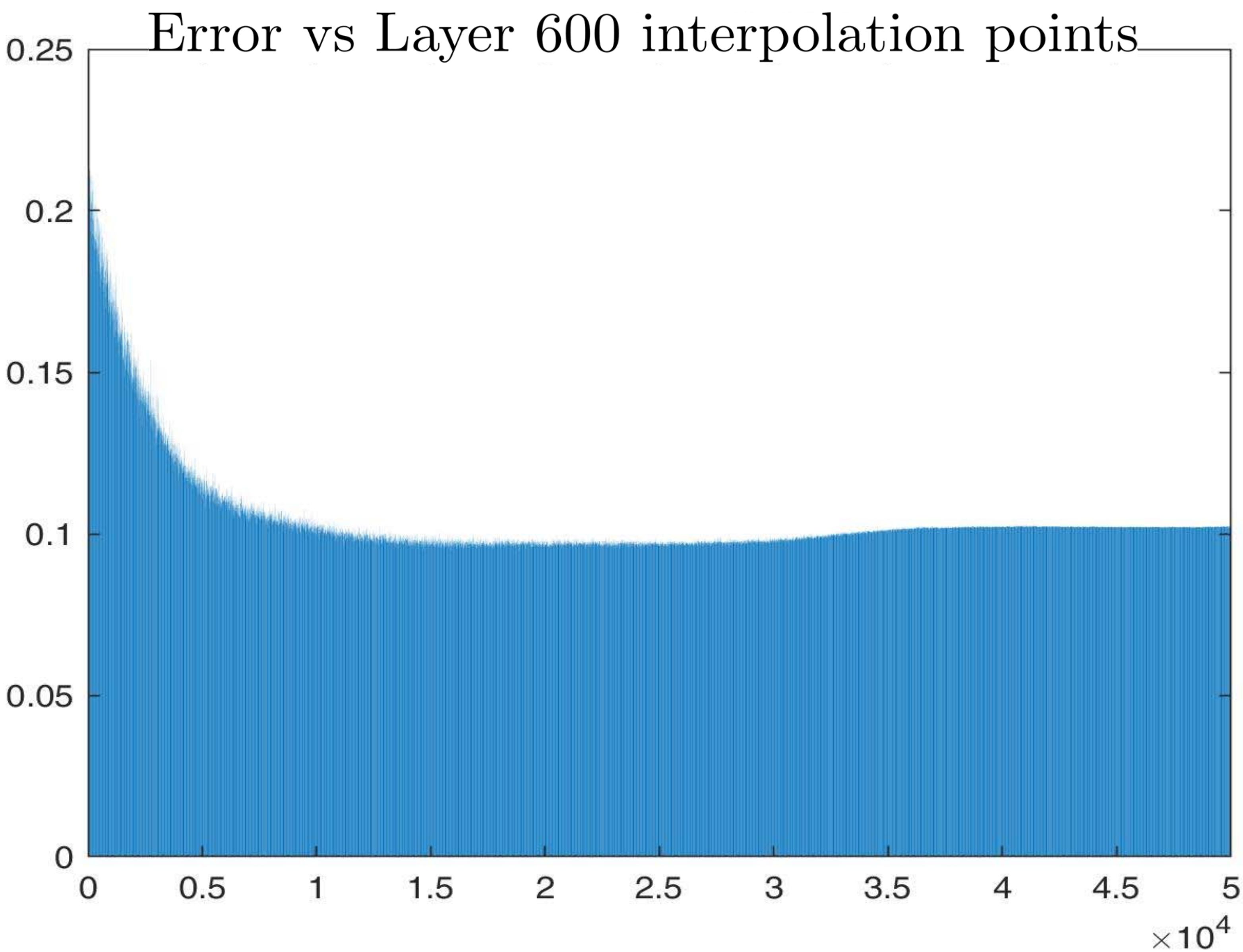
Fashion-MNIST Test Error vs layer



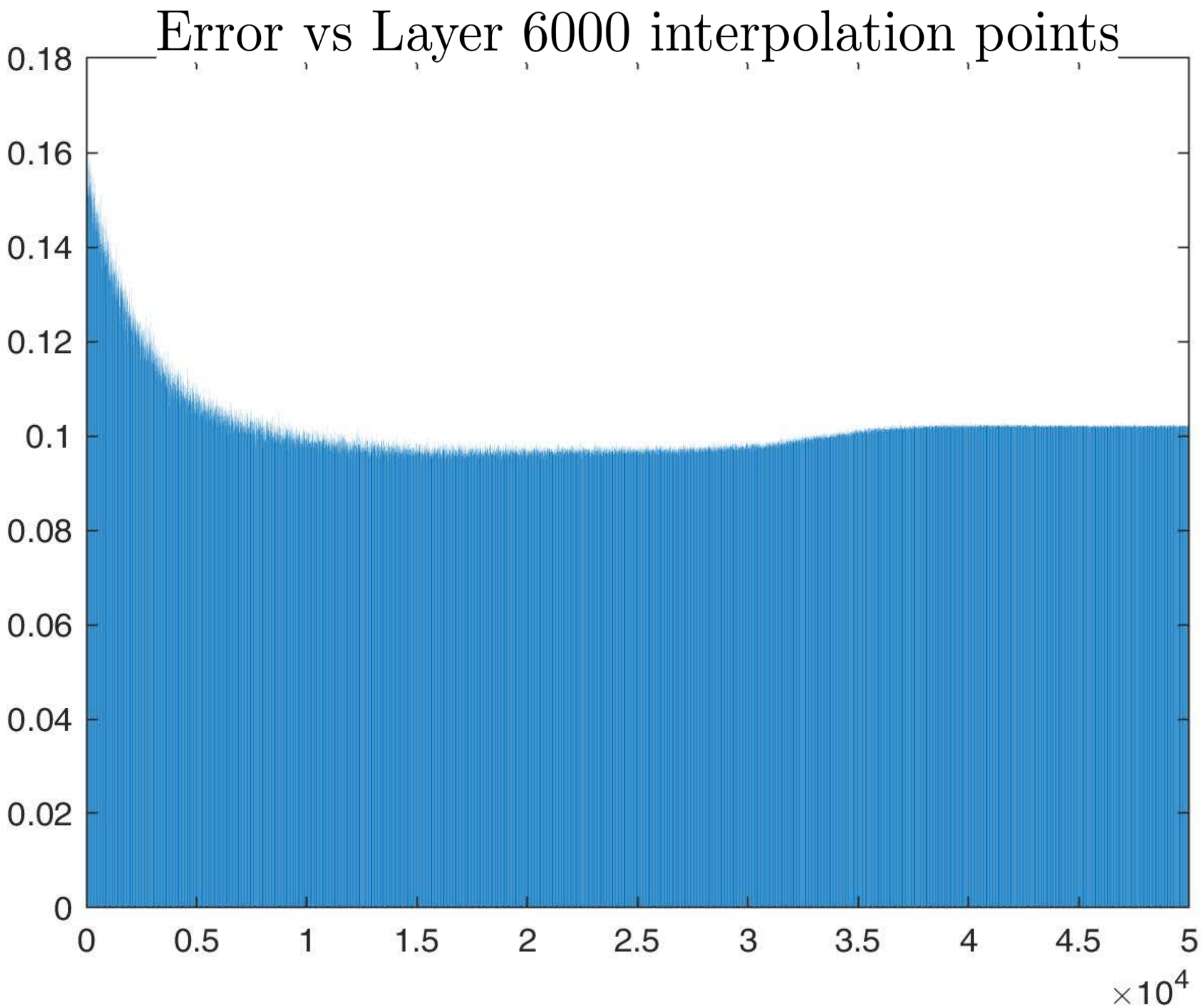
Fashion-MNIST Test Error vs layer



Fashion-MNIST Test Error vs layer



Fashion-MNIST Test Error vs layer



Fashion MNIST

For $15000 \leq n \leq 25000$

9.7% average error with K_n and 600 interpolation points

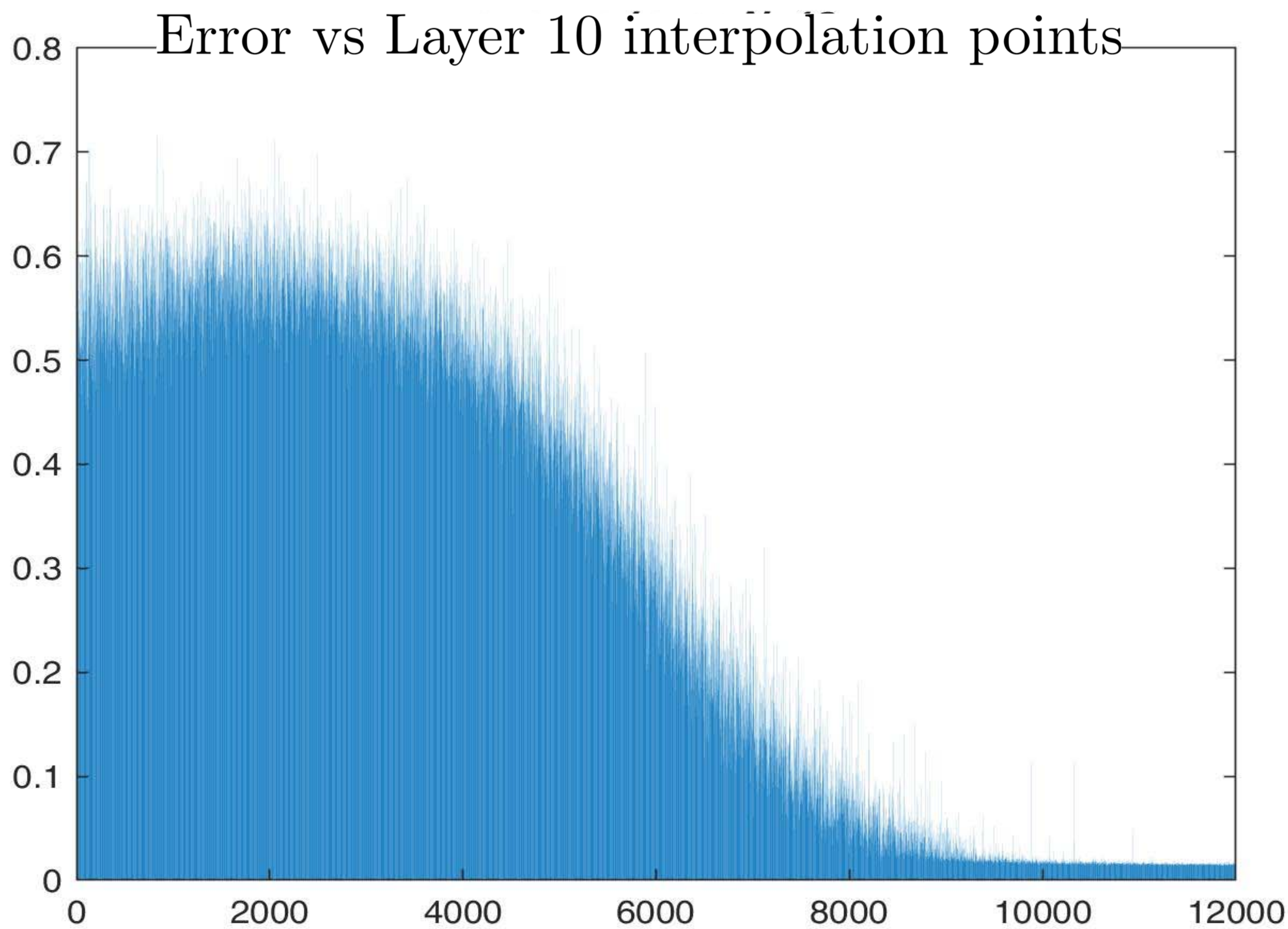
N_I	Average error	Min error	Max error	Standard Deviation
6000	0.0969	0.0944	0.1	7.56×10^{-4}
600	0.0977	0.0951	0.101	8.57×10^{-4}
60	0.114	0.0958	0.22	0.0169
10	0.444	0.15	0.722	0.096

For $49900 \leq n \leq 50000$

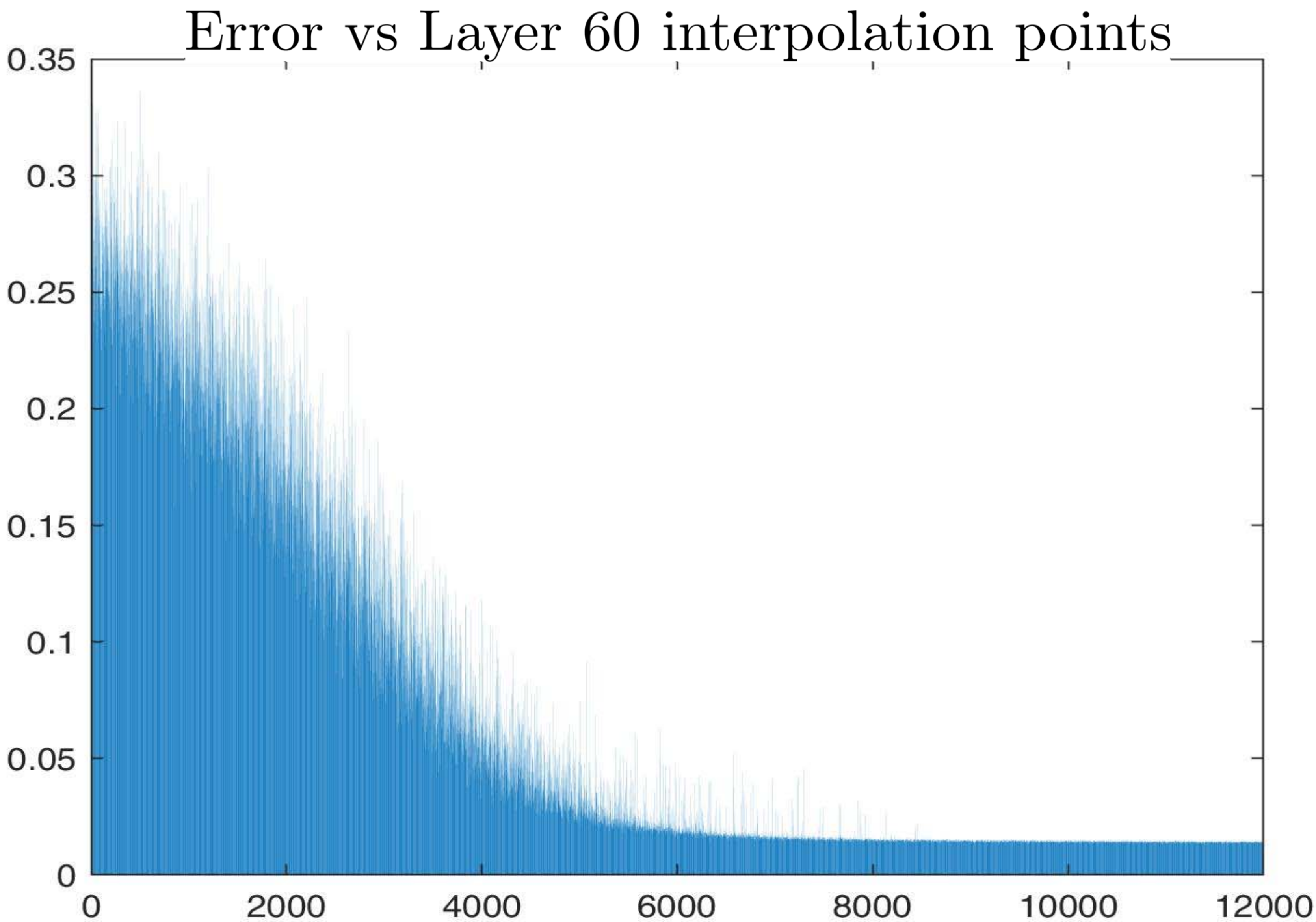
10% average error with K_n and 10 interpolation points

N_I	Average error	Min error	Max error	Standard Deviation
6000	0.10023	0.0999	0.1006	1.6316×10^{-4}
600	0.10013	0.0999	0.1004	1.1671×10^{-4}
60	0.10018	0.0999	0.1005	1.445×10^{-4}
10	0.10018	0.0996	0.1009	2.2941×10^{-4}

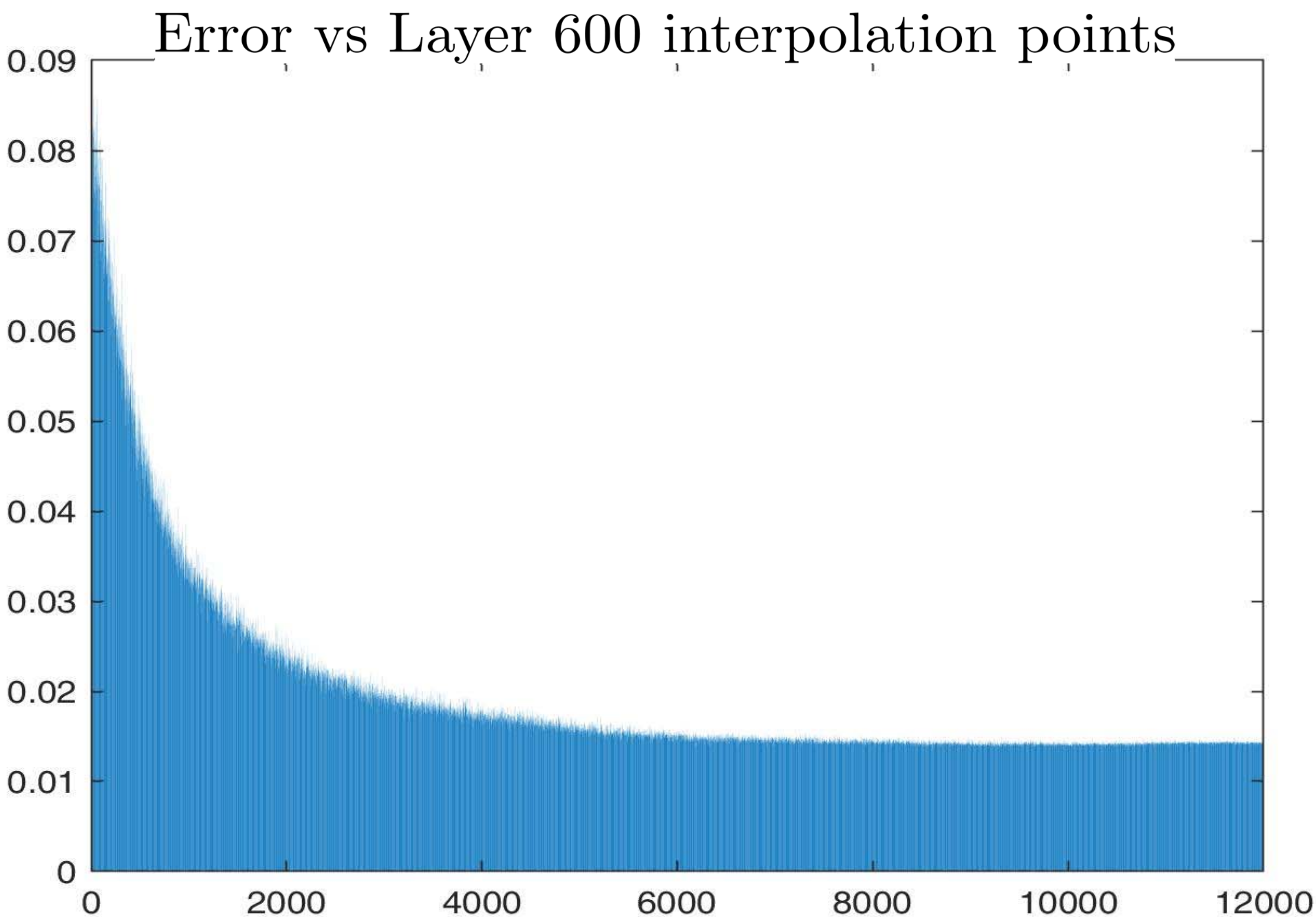
MNIST Test Error vs layer



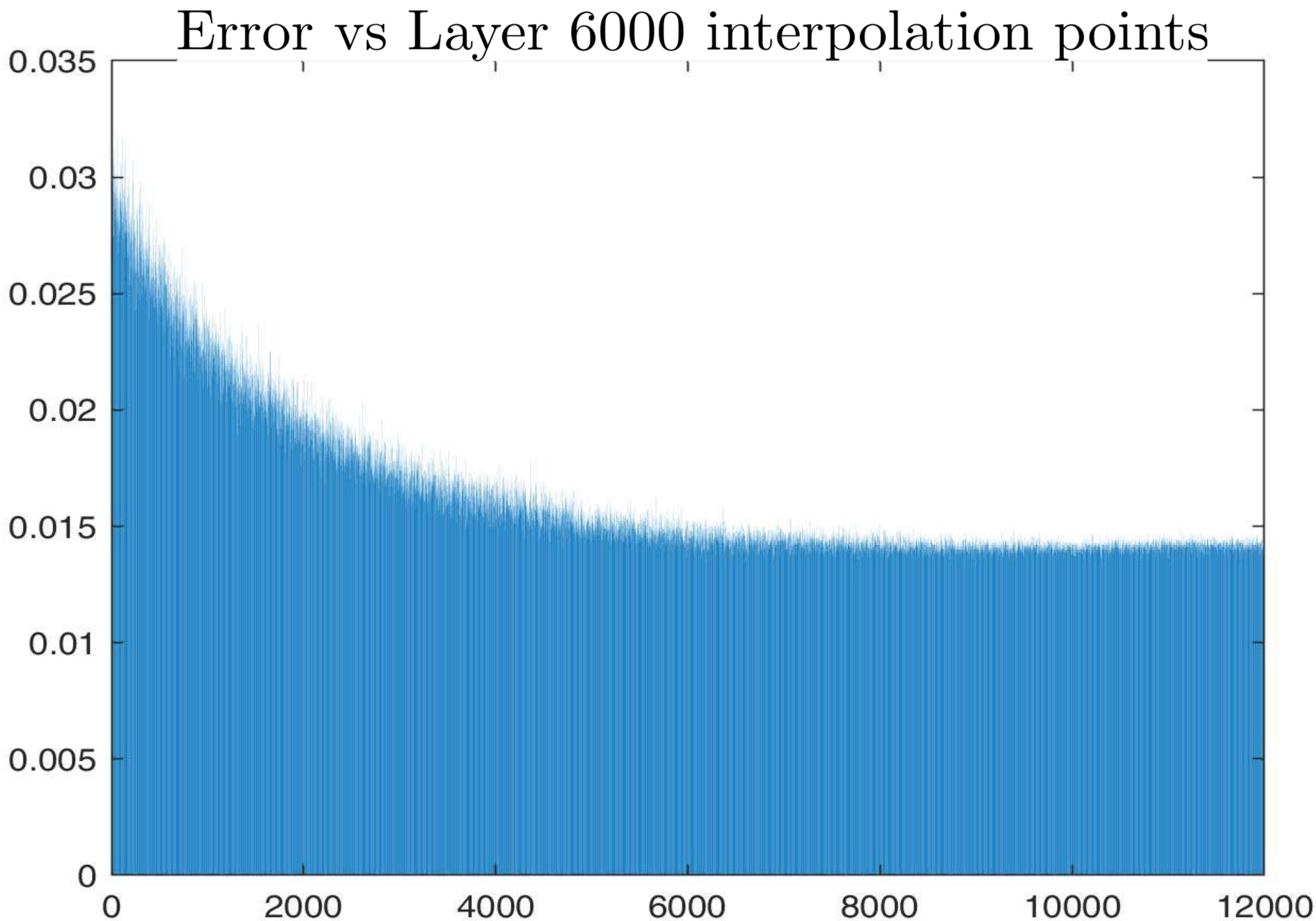
MNIST Test Error vs layer



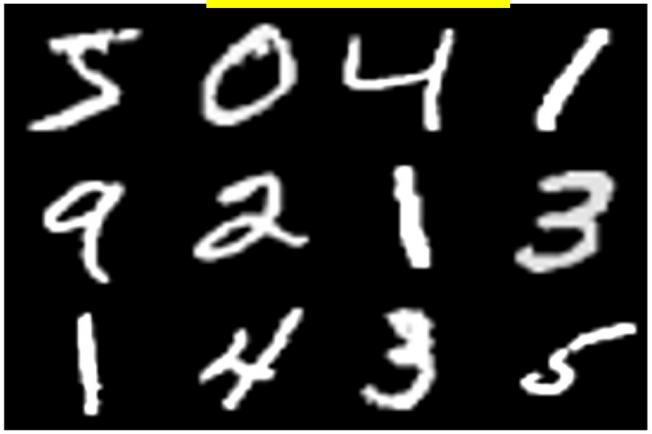
MNIST Test Error vs layer



MNIST Test Error vs layer



MNIST



$N = 60000$

10000 test points

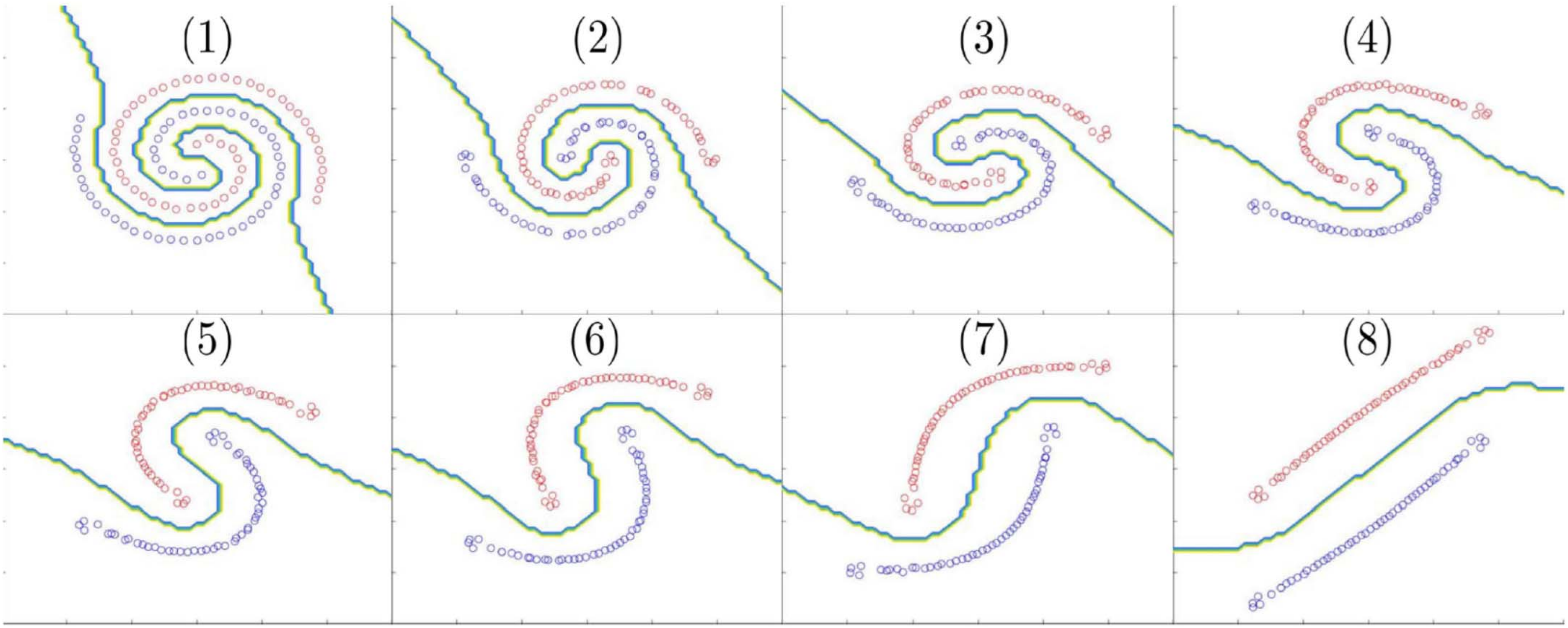
$N_f = 600$

$n = 12000$

1.5% average error with K_n and 10 interpolation points

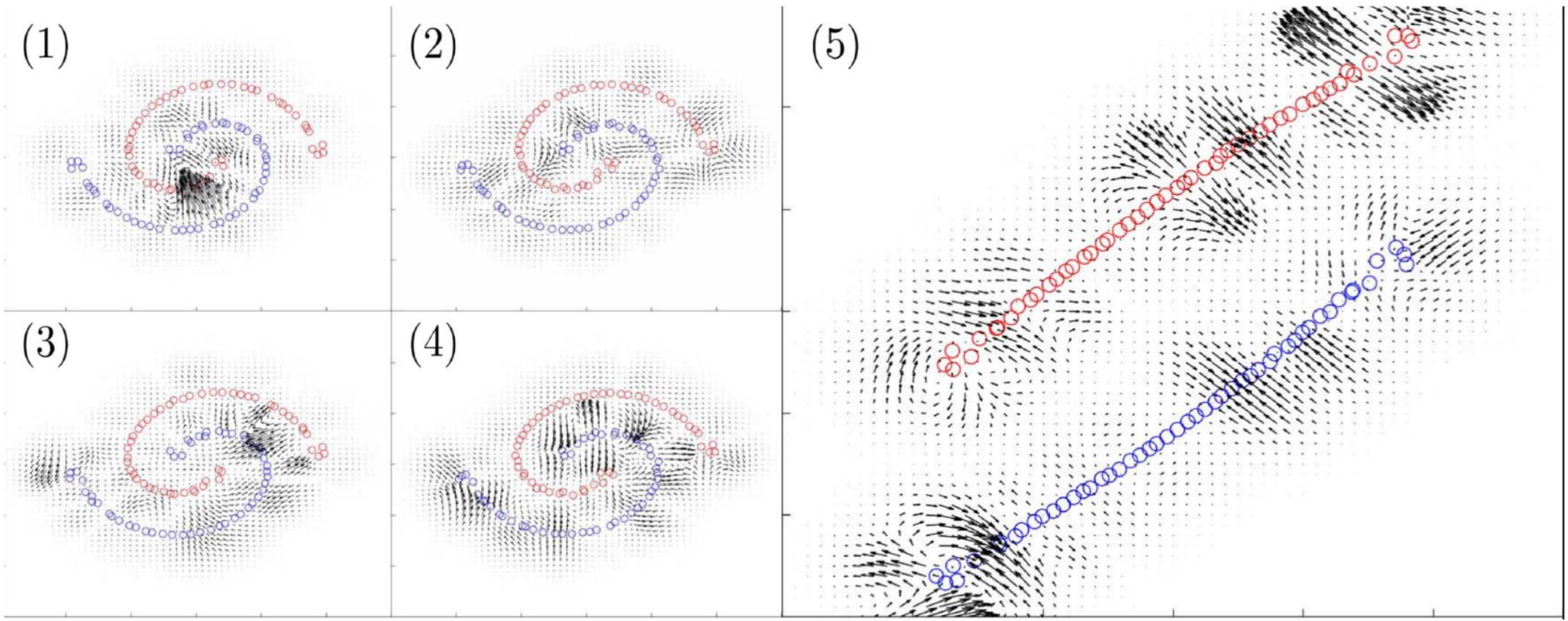
N_I	Average error	Min error	Max error	Standard Deviation
6000	0.014	0.0136	0.0143	1.44×10^{-4}
600	0.014	0.0137	0.0142	9.79×10^{-5}
60	0.0141	0.0136	0.0146	2.03×10^{-4}
10	0.015	0.0136	0.0177	7.13×10^{-4}

$F_n(x_i)$, Gaussian kernel + nugget, $\epsilon = 0.2$



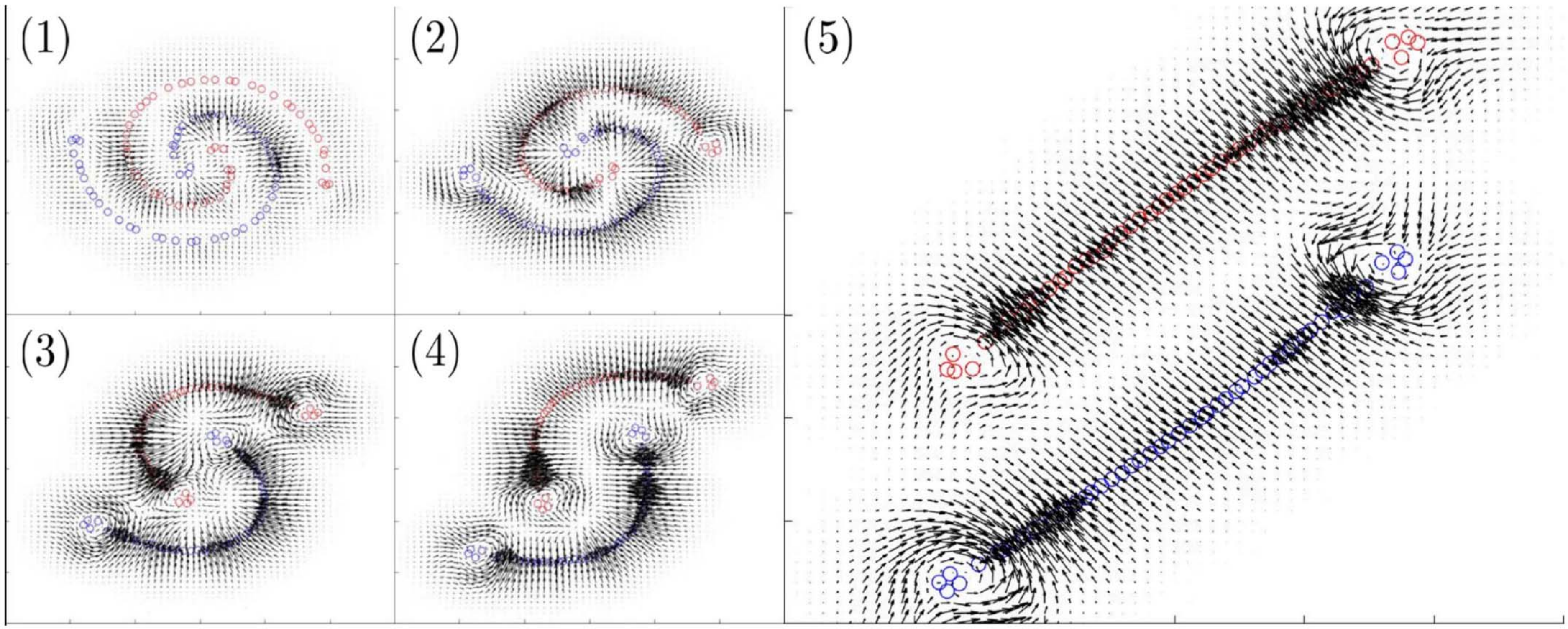
Instantaneous velocity field

$$F_{n+1}(x) - F_n(x)$$



Average velocity field

$$10(F_{n+300}(x) - F_n(x))/300$$



The effective dynamical system

$$\text{As } \epsilon \downarrow 0,\ F_{\text{round}(\frac{t}{\epsilon})}(x) \to F(t,x)$$

$$\boxed{\frac{\partial F(t,x)}{\partial t} = -\mathbb{E}_{X,\pi}\left[\left(\left(\nabla_Z \rho(X,Z,\pi)\right)^T \left(K_1(Z,Z)\right)^{-1} K_1(Z,x)\right)\Big|_{Z=F(X,t)}\right]}$$

$$\boxed{\rho(X,Z,\pi) = 1 - \frac{u(X)^T \pi^T (K_1(\pi Z,\pi Z))^{-1} \pi u(X)}{u(X)^T (K_1(Z,Z))^{-1} u(X)}}$$

- X: random vector of $\mathcal{X}^{N_f}$ representing the random sampling of the training data in a batch size N_f
- $u(X) \in \mathcal{Y}^{N_f}$ is the vector whose entries are the labels of the entries of $X \in \mathcal{X}^{N_f}$
- π: Random $N_c \times N_f$ matrix corresponding to the selection of N_c elements at out N_f (at random, uniformly, without replacement)

Thank you