

# FUNCTIONAL DEPENDENCY THEORY II

CS121: Relational Databases  
Fall 2018 – Lecture 20

# Canonical Cover

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- A canonical cover  $F_c$  for  $F$  is a set of functional dependencies such that:
  - $F$  logically implies all dependencies in  $F_c$
  - $F_c$  logically implies all dependencies in  $F$
  - Can't infer any functional dependency in  $F_c$  from other dependencies in  $F_c$
  - No functional dependency in  $F_c$  contains an extraneous attribute
  - Left side of all functional dependencies in  $F_c$  are unique
    - There are no two dependencies  $\alpha_1 \rightarrow \beta_1$  and  $\alpha_2 \rightarrow \beta_2$  in  $F_c$  such that  $\alpha_1 = \alpha_2$

# Extraneous Attributes

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- Given a set  $F$  of functional dependencies
  - An attribute in a functional dependency is extraneous if it can be removed from  $F$  without changing  $F^+$
- Formally: given  $F$ , and  $\alpha \rightarrow \beta$ 
  - If  $A \in \alpha$ , and  $F$  logically implies  $(F - \{\alpha \rightarrow \beta\}) \cup \{(\alpha - A) \rightarrow \beta\}$ , then  $A$  is extraneous
  - If  $A \in \beta$ , and  $(F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$  logically implies  $F$ , then  $A$  is extraneous
    - i.e. generate a new set of functional dependencies  $F'$  by replacing  $\alpha \rightarrow \beta$  with  $\alpha \rightarrow (\beta - A)$
    - See if  $F'$  logically implies  $F$

# Testing Extraneous Attributes

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- Given relation schema  $R$ , and a set  $F$  of functional dependencies that hold on  $R$
- Attribute  $A$  in  $\alpha \rightarrow \beta$
- If  $A \in \alpha$  (i.e.  $A$  is on left side of the dependency), then let  $\gamma = \alpha - \{A\}$ 
  - ▣ See if  $\gamma \rightarrow \beta$  can be inferred from  $F$
  - ▣ Compute  $\gamma^+$  under  $F$
  - ▣ If  $\beta \subseteq \gamma^+$  then  $A$  is extraneous in  $\alpha$
- e.g. if  $AB \rightarrow C$  and you want to see if  $B$  is extraneous, can see if you can infer  $A \rightarrow C$  from  $F$

# Testing Extraneous Attributes (2)

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- Given relation schema  $R$ , and a set  $F$  of functional dependencies that hold on  $R$
- Attribute  $A$  in  $\alpha \rightarrow \beta$
- If  $A \in \beta$  (on right side of the dependency), then try the altered set  $F'$ 
  - ▣  $F' = (F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$
  - ▣ See if  $\alpha \rightarrow A$  can be inferred from  $F'$
  - ▣ Compute  $\alpha^+$  under  $F'$
  - ▣ If  $\alpha^+$  includes  $A$  then  $A$  is extraneous in  $\beta$
- e.g. if  $A \rightarrow BC$  and you want to see if  $B$  is extraneous, you can already infer  $A \rightarrow B$  from this dependency
  - ▣ Must generate  $F'$  with only  $A \rightarrow C$ , and if you can infer  $A \rightarrow B$  from  $F'$ , then  $B$  was indeed extraneous

# Computing Canonical Cover

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- A simple way to compute the canonical cover of  $F$

$$F_c = F$$

**repeat**

apply union rule to replace dependencies in  $F_c$  of form

$$\alpha_1 \rightarrow \beta_1 \text{ and } \alpha_1 \rightarrow \beta_2 \text{ with } \alpha_1 \rightarrow \beta_1\beta_2$$

find a functional dependency  $\alpha \rightarrow \beta$  in  $F_c$  with an  
extraneous attribute

/\* Use  $F_c$  for the extraneous attribute test, not  $F$  !!! \*/

if an extraneous attribute is found, delete it from  $\alpha \rightarrow \beta$

**until**  $F_c$  stops changing

# Canonical Cover Example

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- Functional dependencies  $F$  on schema  $(A, B, C)$ 
  - $F = \{ A \rightarrow BC, B \rightarrow C, A \rightarrow B, AB \rightarrow C \}$
  - Find  $F_c$
- Apply union rule to  $A \rightarrow BC$  and  $A \rightarrow B$ 
  - Left with:  $\{ A \rightarrow BC, B \rightarrow C, AB \rightarrow C \}$
- $A$  is extraneous in  $AB \rightarrow C$ 
  - $B \rightarrow C$  is logically implied by  $F$  (obvious)
  - Left with:  $\{ A \rightarrow BC, B \rightarrow C \}$
- $C$  is extraneous in  $A \rightarrow BC$ 
  - Logically implied by  $A \rightarrow B, B \rightarrow C$
- $F_c = \{ A \rightarrow B, B \rightarrow C \}$

# Canonical Covers

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- A set of functional dependencies can have multiple canonical covers
- Example:
  - $F = \{ A \rightarrow BC, B \rightarrow AC, C \rightarrow AB \}$
  - Has several canonical covers:
    - $F_c = \{ A \rightarrow B, B \rightarrow C, C \rightarrow A \}$
    - $F_c = \{ A \rightarrow B, B \rightarrow AC, C \rightarrow B \}$
    - $F_c = \{ A \rightarrow C, C \rightarrow B, B \rightarrow A \}$
    - $F_c = \{ A \rightarrow C, B \rightarrow C, C \rightarrow AB \}$
    - $F_c = \{ A \rightarrow BC, B \rightarrow A, C \rightarrow A \}$



# Another Example

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- Functional dependencies  $F$  on schema  $(A, B, C, D)$ 
  - $F = \{ A \rightarrow B, BC \rightarrow D, AC \rightarrow D \}$
  - Find  $F_c$
- In this case, it may look like  $F_c = F \dots$
- However, can infer  $AC \rightarrow D$  from  $A \rightarrow B, BC \rightarrow D$  (pseudotransitivity), so  $AC \rightarrow D$  is extraneous in  $F$ 
  - Therefore,  $F_c = \{ A \rightarrow B, BC \rightarrow D \}$
- Alternately, can argue that  $D$  is extraneous in  $AC \rightarrow D$ 
  - With  $F' = \{ A \rightarrow B, BC \rightarrow D \}$ , we see that  $\{AC\}^+ = ABCD$ , so  $D$  is extraneous in  $AC \rightarrow D$
  - (If you eliminate the entire RHS of a functional dependency, it goes away)

# Lossy Decompositions

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- Some schema decompositions lose information

- Example:

*employee*(*emp\_id*, *emp\_name*, *phone*, *title*, *salary*, *start\_date*)

- ▣ Decomposed into:

*emp\_ids*(*emp\_id*, *emp\_name*)

*emp\_details*(*emp\_name*, *phone*, *title*, *salary*, *start\_date*)

- Problem:

- ▣ *emp\_name* doesn't uniquely identify employees
- ▣ This is a lossy decomposition

# Lossless Decompositions

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- Given:
  - ▣ Relation schema  $R$ , relation  $r(R)$
  - ▣ Set of functional dependencies  $F$
- Let  $R_1$  and  $R_2$  be a decomposition of  $R$ 
  - ▣  $R_1 \cup R_2 = R$
- The decomposition is lossless if, for all legal instances of  $r$  :
$$\Pi_{R_1}(r) \bowtie \Pi_{R_2}(r) = r$$
- A simple definition...

# Lossless Decompositions (2)

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- Can define with functional dependencies:
  - ▣  $R_1$  and  $R_2$  form a lossless decomposition of  $R$  if at least one of these dependencies is in  $F^+$  :

$$R_1 \cap R_2 \rightarrow R_1$$

$$R_1 \cap R_2 \rightarrow R_2$$

- $R_1 \cap R_2$  forms a superkey of  $R_1$  and/or  $R_2$ 
  - ▣ Test for superkeys using attribute-set closure

# Decomposition Examples (1)

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- The *employee* example:

*employee*(*emp\_id*, *emp\_name*, *phone*, *title*, *salary*,  
*start\_date*)

- Decomposed into:

*emp\_ids*(*emp\_id*, *emp\_name*)

*emp\_details*(*emp\_name*, *phone*, *title*, *salary*, *start\_date*)

- *emp\_name* is not a superkey of *emp\_ids* or *emp\_details*, so the decomposition is lossy

# Decomposition Examples (2)

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- The *bor\_loan* example:

*bor\_loan*(*cust\_id*, *loan\_id*, *amount*)

- Decomposed into:

*borrower*(*cust\_id*, *loan\_id*)

*loan*(*loan\_id*, *amount*)     ( *loan\_id* → *loan\_id*, *amount* )

- *loan\_id* is a superkey of *loan*, so the decomposition is lossless

# BCNF Decompositions

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- If  $R$  is a schema not in BCNF:
  - ▣ There is at least one nontrivial functional dependency  $\alpha \rightarrow \beta$  such that  $\alpha$  is not a superkey for  $R$
  - ▣ For simplicity, also require that  $\alpha \cap \beta = \emptyset$ 
    - (if  $\alpha \cap \beta \neq \emptyset$  then  $(\alpha \cap \beta)$  is extraneous in  $\beta$ )
- Replace  $R$  with two schemas:
  - $R_1 = (\alpha \cup \beta)$
  - $R_2 = (R - \beta)$ 
    - (was  $R - (\beta - \alpha)$ , but  $\beta - \alpha = \beta$ , since  $\alpha \cap \beta = \emptyset$ )
- BCNF decomposition is lossless
  - ▣  $R_1 \cap R_2 = \alpha$
  - ▣  $\alpha$  is a superkey of  $R_1$
  - ▣  $\alpha$  also appears in  $R_2$

# Dependency Preservation

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- Some schema decompositions are not dependency-preserving
  - ▣ Functional dependencies that span multiple relation schemas are hard to enforce
  - ▣ e.g. BCNF may require decomposition of a schema for one dependency, and make it hard to enforce another dependency
- Can test for dependency preservation using functional dependency theory



# Dependency Preservation (2)

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- Given:
  - A set  $F$  of functional dependencies on a schema  $R$
  - $R_1, R_2, \dots, R_n$  are a decomposition of  $R$
- The restriction of  $F$  to  $R_i$  is the set  $F_i$  of functional dependencies in  $F^+$  that only has attributes in  $R_i$ 
  - Each  $F_i$  contains functional dependencies that can be checked efficiently, using only  $R_i$
- Find *all* functional dependencies that can be checked efficiently
  - $F' = F_1 \cup F_2 \cup \dots \cup F_n$
  - If  $F'^+ = F^+$  then the decomposition is dependency-preserving

# Third Normal Form Schemas

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- Can generate a 3NF schema from a set of functional dependencies  $F$
- Called the 3NF synthesis algorithm
  - ▣ Instead of decomposing an initial schema, generates schemas from a set of dependencies
- Given a set  $F$  of functional dependencies
  - ▣ Uses the canonical cover  $F_c$
  - ▣ Ensures that resulting schemas are dependency-preserving

# 3NF Synthesis Algorithm

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□ Inputs: set of functional dependencies  $F$ , on a schema  $R$

let  $F_c$  be a canonical cover for  $F$  ;

$i := 0$ ;

**for each** functional dependency  $\alpha \rightarrow \beta$  in  $F_c$  **do**

**if** none of the schemas  $R_j, j = 1, 2, \dots, i$  contains  $(\alpha \cup \beta)$  **then**

$i := i + 1$ ;

$R_i := (\alpha \cup \beta)$

**end if**

**done**

**if** no schema  $R_j, j = 1, 2, \dots, i$  contains a candidate key for  $R$  **then**

$i := i + 1$ ;

$R_i :=$  any candidate key for  $R$

**end if**

**return**  $(R_1, R_2, \dots, R_i)$

# BCNF vs. 3NF

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- Boyce-Codd Normal Form:
  - ▣ Eliminates more redundant information than 3NF
  - ▣ Some functional dependencies become expensive to enforce
    - The conditions to enforce involve multiple relations
  - ▣ Overall, a very desirable normal form!
- Third Normal Form:
  - ▣ All [more] dependencies are [probably] easy to enforce...
  - ▣ Allows more redundant information, which must be kept synchronized by the database application!
  - ▣ Personal banker example:
    - works\_in(emp\_id, branch\_name)*
    - cust\_banker\_branch(cust\_id, branch\_name, emp\_id, type)*
    - Branch names must be kept synchronized between these relations!

# BCNF and 3NF vs. SQL

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- SQL constraints:
  - ▣ Only key constraints are fast and easy to enforce!
  - ▣ Only easy to enforce functional dependencies  $\alpha \rightarrow \beta$  if  $\alpha$  is a key on some table!
  - ▣ Other functional dependencies (even “easy” ones in 3NF) may require more expensive constraints, e.g. **CHECK**
- For SQL databases with materialized views:
  - ▣ Can decompose a schema into BCNF
  - ▣ For dependencies  $\alpha \rightarrow \beta$  not preserved in decomposition, create materialized view joining all relations in dependency
  - ▣ Enforce **unique**( $\alpha$ ) constraint on materialized view
- Impacts both space and performance, but it works...

# Multivalued Attributes

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- E-R schemas can have multivalued attributes
- 1NF requires only atomic attributes
  - ▣ Not a problem; translating to relational model leaves everything atomic

- Employee example:

*employee*(*emp\_id*, *emp\_name*)

*emp\_deps*(*emp\_id*, *dependent*)

*emp\_nums*(*emp\_id*, *phone\_num*)

<i>employee</i>
<u><i>emp_id</i></u>
<i>emp_name</i>
{ <i>phone_num</i> }
{ <i>dependent</i> }

- What are the requirements on these schemas for what tuples must appear?

# Multivalued Attributes (2)

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## □ Example data:

<i>emp_id</i>	<i>emp_name</i>
125623	Rick

*employee*

<i>emp_id</i>	<i>dependent</i>
125623	Jeff
125623	Alice

*emp\_deps*

<i>emp_id</i>	<i>phone_num</i>
125623	555-8888
125623	555-2222

*emp\_nums*

- Every distinct value of multivalued attribute requires a separate tuple, including associated value of *emp\_id*
- A consequence of 1NF, in fact!
  - If attributes could be nonatomic, could just store list of values in the appropriate column!
  - 1NF requires extra tuples to represent multivalues

# Independent Multivalued Attributes

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- Question is trickier when a schema stores several *independent* multivalued attributes
- Proposed combined schema:  
*employee(emp\_id, emp\_name)*  
*emp\_info(emp\_id, dependent, phone\_num)*
- What tuples must appear in *emp\_info* ?
  - ▣ *emp\_info* is a relation
  - ▣ If an employee has  $M$  dependents and  $N$  phone numbers, *emp\_info* must contain  $M \times N$  tuples
    - Exactly what we get if we natural-join *emp\_deps* and *emp\_nums*
  - ▣ Every combination of the employee's dependents and their phone numbers



# Independent Multivalued Attributes

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- Example data:

<i>emp_id</i>	<i>emp_name</i>
125623	Rick

*employee*

<i>emp_id</i>	<i>dependent</i>	<i>phone_num</i>
125623	Jeff	555-8888
125623	Jeff	555-2222
125623	Alice	555-8888
125623	Alice	555-2222

*emp\_info*

- Clearly has unnecessary redundancy
- Can't formulate functional dependencies to represent multivalued attributes
- Can't use BCNF or 3NF decompositions to eliminate redundancy in these cases

# Multivalued Attributes Example

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- Two employees: Rick and Bob
  - ▣ Both share a phone number at work
  - ▣ Both have two kids
  - ▣ Both have a kid named Alice
- Can't use functional dependencies to reason about this situation!
  - ▣  $emp\_id \rightarrow phone\_num$  doesn't hold since an employee can have several phone numbers
  - ▣  $phone\_num \rightarrow emp\_id$  doesn't hold either, since several employees can have the same phone number
  - ▣ Same with  $emp\_id$  and  $dependent$ ...

<i>emp_id</i>	<i>emp_name</i>
125623	Rick
127341	Bob

*employee*

<i>emp_id</i>	<i>phone_num</i>
125623	555-8888
125623	555-2222
127341	555-2222

*emp\_nums*

<i>emp_id</i>	<i>dependent</i>
125623	Jeff
125623	Alice
127341	Alice
127341	Clara

*emp\_deps*

# Dependencies

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- Functional dependencies rule out what tuples can appear in a relation
  - ▣ If  $A \rightarrow B$  holds, then tuples cannot have same value for  $A$  but different values for  $B$
  - ▣ Also called equality-generating dependencies
- Multivalued dependencies specify what tuples must be present
  - ▣ To represent a multivalued attribute's values properly, a certain set of tuples *must* be present
  - ▣ Also called tuple-generating dependencies

# Multivalued Dependencies

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- Given a relation schema  $R$ 
  - Attribute-sets  $\alpha \in R, \beta \in R$
  - $\alpha \twoheadrightarrow \beta$  is a multivalued dependency
  - “ $\alpha$  multidetermines  $\beta$ ”
- A multivalued dependency  $\alpha \twoheadrightarrow \beta$  holds on  $R$  if, in any legal relation  $r(R)$ :

For all pairs of tuples  $t_1$  and  $t_2$  in  $r$  such that  $t_1[\alpha] = t_2[\alpha]$ ,  
There also exists tuples  $t_3$  and  $t_4$  in  $r$  such that:

  - $t_1[\alpha] = t_2[\alpha] = t_3[\alpha] = t_4[\alpha]$
  - $t_1[\beta] = t_3[\beta]$  and  $t_2[\beta] = t_4[\beta]$
  - $t_1[R - \beta] = t_4[R - \beta]$  and  $t_2[R - \beta] = t_3[R - \beta]$

# Multivalued Dependencies (2)

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- Multivalued dependency  $\alpha \twoheadrightarrow \beta$  holds on  $R$  if, in any legal relation  $r(R)$ :

For all pairs of tuples  $t_1$  and  $t_2$  in  $r$  such that  $t_1[\alpha] = t_2[\alpha]$ ,

There also exists tuples  $t_3$  and  $t_4$  in  $r$  such that:

- $t_1[\alpha] = t_2[\alpha] = t_3[\alpha] = t_4[\alpha]$
  - $t_1[\beta] = t_3[\beta]$  and  $t_2[\beta] = t_4[\beta]$
  - $t_1[R - \beta] = t_4[R - \beta]$  and  $t_2[R - \beta] = t_3[R - \beta]$
- Pictorially:

	$\alpha$	$\beta$	$R - (\alpha \cup \beta)$
$t_1$	$a_1 \dots a_i$	$a_{i+1} \dots a_j$	$a_{j+1} \dots a_n$
$t_2$	$a_1 \dots a_i$	$b_{i+1} \dots b_j$	$b_{j+1} \dots b_n$
$t_3$	$a_1 \dots a_i$	$a_{i+1} \dots a_j$	$b_{j+1} \dots b_n$
$t_4$	$a_1 \dots a_i$	$b_{i+1} \dots b_j$	$a_{j+1} \dots a_n$

# Multivalued Dependencies (3)

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- Multivalued dependency:

	$\alpha$	$\beta$	$R - (\alpha \cup \beta)$
$t_1$	$a_1 \dots a_i$	$a_{i+1} \dots a_j$	$a_{j+1} \dots a_n$
$t_2$	$a_1 \dots a_i$	$b_{i+1} \dots b_j$	$b_{j+1} \dots b_n$
$t_3$	$a_1 \dots a_i$	$a_{i+1} \dots a_j$	$b_{j+1} \dots b_n$
$t_4$	$a_1 \dots a_i$	$b_{i+1} \dots b_j$	$a_{j+1} \dots a_n$

- If  $\alpha \twoheadrightarrow \beta$  then  $R - (\alpha \cup \beta)$  is independent of this fact
  - ▣ Every distinct value of  $\beta$  must be associated once with every distinct value of  $R - (\alpha \cup \beta)$
- Let  $\gamma = R - (\alpha \cup \beta)$ 
  - ▣ If  $\alpha \twoheadrightarrow \beta$  then also  $\alpha \twoheadrightarrow \gamma$
  - ▣  $\alpha \twoheadrightarrow \beta$  implies  $\alpha \twoheadrightarrow \gamma$
  - ▣ Sometimes written  $\alpha \twoheadrightarrow \beta \mid \gamma$

# Trivial Multivalued Dependencies

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- $\alpha \twoheadrightarrow \beta$  is a trivial multivalued dependency on  $R$  if all relations  $r(R)$  satisfy the dependency
- Specifically,  $\alpha \twoheadrightarrow \beta$  is trivial if  $\beta \subseteq \alpha$ , or if  $\alpha \cup \beta = R$
- Employee examples:
  - ▣ For schema  $emp\_deps(emp\_id, dependent)$ ,  $emp\_id \twoheadrightarrow dependent$  is trivial
  - ▣ For  $emp\_info(emp\_id, dependent, phone\_num)$ ,  $emp\_id \twoheadrightarrow dependent$  is not trivial

# Inference Rules

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- Can reason about multivalued dependencies, just like functional dependencies
  - There is a set of complete, sound inference rules for MVDs
- Example inference rules:
  - Complementation rule:
    - If  $\alpha \twoheadrightarrow \beta$  holds on  $R$ , then  $\alpha \twoheadrightarrow R - (\alpha \cup \beta)$  holds
  - Multivalued augmentation rule:
    - If  $\alpha \twoheadrightarrow \beta$  holds, and  $\gamma \subseteq R$ , and  $\delta \subseteq \gamma$ , then  $\gamma\alpha \twoheadrightarrow \delta\beta$  holds
  - Multivalued transitivity rule:
    - If  $\alpha \twoheadrightarrow \beta$  and  $\beta \twoheadrightarrow \gamma$  holds, then  $\alpha \twoheadrightarrow \gamma - \beta$  holds
  - Coalescence rule:
    - If  $\alpha \twoheadrightarrow \beta$  holds, and  $\gamma \subseteq \beta$ , and there is a  $\delta$  such that  $\delta \subseteq R$ , and  $\delta \cap \beta = \emptyset$ , and  $\delta \twoheadrightarrow \gamma$ , then  $\alpha \twoheadrightarrow \gamma$  holds



# Functional Dependencies

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- Functional dependencies are also multivalued dependencies
- Replication rule:
  - ▣ If  $\alpha \rightarrow \beta$ , then  $\alpha \twoheadrightarrow \beta$  too
  - ▣ Note there is an additional constraint from  $\alpha \rightarrow \beta$ : each value of  $\alpha$  has *at most one* associated value for  $\beta$
- Usually, functional dependencies are not stated as multivalued dependencies
  - ▣ The extra caveat is *important*, but not obvious in notation
  - ▣ Also, functional dependencies are easier to reason about!

# Closures and Restrictions

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- For a set  $D$  of functional and multivalued dependencies, can compute closure  $D^+$ 
  - ▣ Use inference rules for both functional and multivalued dependencies to compute closure
- Sometimes need the restriction of  $D^+$  to a relation schema  $R$ , too
- The restriction of  $D$  to a schema  $R_i$  includes:
  - ▣ All functional dependencies in  $D^+$  that include only attributes in  $R_i$
  - ▣ All multivalued dependencies of the form  $\alpha \twoheadrightarrow \beta \cap R_i$ , where  $\alpha \subseteq R_i$ , and  $\alpha \twoheadrightarrow \beta$  is in  $D^+$

# Fourth Normal Form

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- Given:
  - ▣ Relation schema  $R$
  - ▣ Set of functional and multivalued dependencies  $D$
- $R$  is in 4NF with respect to  $D$  if:
  - ▣ For all multivalued dependencies  $\alpha \twoheadrightarrow \beta$  in  $D^+$ , where  $\alpha \subseteq R$  and  $\beta \subseteq R$ , at least one of the following holds:
    - $\alpha \twoheadrightarrow \beta$  is a trivial multivalued dependency
    - $\alpha$  is a superkey for  $R$
  - ▣ Note: If  $\alpha \rightarrow \beta$  then  $\alpha \twoheadrightarrow \beta$
- A database design is in 4NF if all schemas in the design are in 4NF

# 4NF and BCNF

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- Main difference between 4NF and BCNF is use of multivalued dependencies instead of functional dependencies
- Every schema in 4NF is also in BCNF
  - ▣ If a schema is not in BCNF then there is a nontrivial functional dependency  $\alpha \rightarrow \beta$  such that  $\alpha$  is not a superkey for  $R$
  - ▣ If  $\alpha \rightarrow \beta$  then  $\alpha \twoheadrightarrow \beta$

# 4NF Decompositions

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- Decomposition rule is very similar to BCNF
- If schema  $R$  is not in 4NF with respect to a set of multivalued dependencies  $D$  :
  - ▣ There is some nontrivial dependency  $\alpha \twoheadrightarrow \beta$  in  $D^+$  where  $\alpha \subseteq R$  and  $\beta \subseteq R$ , and  $\alpha$  is not a superkey of  $R$ 
    - Also constrain that  $\alpha \cap \beta = \emptyset$
  - ▣ Replace  $R$  with two new schemas:
    - $R_1 = (\alpha \cup \beta)$
    - $R_2 = (R - \beta)$

# Employee Information Example

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- Combined schema:

*employee*(*emp\_id*, *emp\_name*)

*emp\_info*(*emp\_id*, *dependent*, *phone\_num*)

- Also have these dependencies:

- $emp\_id \rightarrow emp\_name$

- $emp\_id \twoheadrightarrow dependent$

- $emp\_id \twoheadrightarrow phone\_num$

- *emp\_info* is not in 4NF

- Following the rules for 4NF decomposition produces:

*(emp\_id, dependent)*

*(emp\_id, phone\_num)*

- Note: Each relation's candidate key is the entire relation. The multivalued dependencies are trivial.

# Lossless Decompositions

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- Can also define lossless decomposition with multivalued dependencies
  - $R_1$  and  $R_2$  form a lossless decomposition of  $R$  if at least one of these dependencies is in  $D^+$  :

$$R_1 \cap R_2 \twoheadrightarrow R_1$$

$$R_1 \cap R_2 \twoheadrightarrow R_2$$

# Beyond Fourth Normal Form?

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- Additional normal forms with various constraints
- Example: join dependencies
- Given  $R$ , and a decomposition  $R_1$  and  $R_2$  where  $R_1 \cup R_2 = R$  :
  - ▣ The decomposition is lossless if, for all legal instances of  $r(R)$ ,  
 $\Pi_{R_1}(r) \bowtie \Pi_{R_2}(r) = r$
- Can state this as a join dependency:  $*(R_1, R_2)$ 
  - ▣ This is actually identical to a multivalued dependency!
  - ▣  $*(R_1, R_2)$  is equivalent to  $R_1 \cap R_2 \twoheadrightarrow R_1 \mid R_2$



# Join Dependencies and 5NF

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- Join dependencies (JD) are a generalization of multivalued dependencies (MVD)
  - ▣ Can specify JDs involving  $N$  relation schemas,  $N \geq 2$
  - ▣ JDs are equivalent to MVDs when  $N = 2$
  - ▣ Can easily construct JDs where  $N > 2$ , with no equivalent set of MVDs
- Project-Join Normal Form (a.k.a. PJNF or 5NF):
  - ▣ A relation schema  $R$  is in PJNF with respect to a set of join dependencies  $D$  if, for all JDs in  $D^+$  of the form  $*(R_1, R_2, \dots, R_n)$  where  $R_1 \cup R_2 \cup \dots \cup R_n = R$ , at least one of the following holds:
    - $*(R_1, R_2, \dots, R_n)$  is a trivial join dependency
    - Every  $R_i$  is a superkey for  $R$

# Join Dependencies and 5NF (2)

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- If a schema is in Project-Join Normal Form then it is also in 4NF (and thus, in BCNF)
  - ▣ Every multivalued dependency is also a join dependency
  - ▣ (Every functional dependency is also a multivalued dependency)
- One small problem:
  - ▣ There isn't a complete, sound set of inference rules for join dependencies!
  - ▣ Can't reason about our set of join dependencies  $D...$
  - ▣ This limits PJNF's real-world usefulness

# Domain-Key Normal Form

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- Domain-key normal form (DKNF) is an even more general normal form, based on:
  - **Domain constraints:** what values may be assigned to attribute  $A$ 
    - Usually inexpensive to test, even with **CHECK** constraints
  - **Key constraints:** all attribute-sets  $K$  that are a superkey for a schema  $R$  (i.e.  $K \rightarrow R$ )
    - Almost always inexpensive to test
  - **General constraints:** other predicates on valid relations in a schema
    - Could be very expensive to test!
- A schema  $R$  is in DKNF if the domain constraints and key constraints logically imply the general constraints
  - An “ideal” normal form difficult to achieve in practice...