FUNCTIONAL DEPENDENCY THEORY

CS121: Relational Databases

Fall 2018 – Lecture 19

Last Lecture

- Normal forms specify "good schema" patterns
- First normal form (1NF):
  - All attributes must be atomic
  - Easy in relational model, harder/less desirable in SQL
- Boyce-Codd normal form (BCNF):
  - Eliminates redundancy using functional dependencies
  - Given a relation schema *R* and a set of dependencies *F*
  - For all functional dependencies $\alpha \rightarrow \beta$ in F^+, where $\alpha \cup \beta \subseteq R$, at least one of these conditions must hold:
    - $\alpha \rightarrow \beta$ is a trivial dependency
    - α is a superkey for *R*

Last Lecture (2)

- Can convert a schema into BCNF
- If R is a schema not in BCNF:
  - There is at least one nontrivial functional dependency $\alpha \rightarrow \beta \in F^+$ such that α is not a superkey for R
- Replace R with two schemas:

  $(\alpha \cup \beta)$

  $(R - (\beta - \alpha))$
- May need to repeat this decomposition process until all schemas are in BCNF

Functional Dependency Theory

- Important to be able to reason about functional dependencies!
- Main question:
  - What functional dependencies are logically implied by a set *F* of functional dependencies?
- Other useful questions:
  - Which attributes are functionally determined by a particular attribute-set?
  - What *minimal* set of functional dependencies must actually be enforced in a database?
  - Is a particular schema decomposition lossless?
  - Does a decomposition preserve dependencies?

Rules of Inference

- Given a set F of functional dependencies
  - Actual dependencies listed in F may be insufficient for normalizing a schema
  - Must consider all dependencies <u>logically implied</u> by F
- For a relation schema R
  - A functional dependency f on R is logically implied by F on R if every relation instance $r(R)$ that satisfies F also satisfies f
- Example:
  - Relation schema $R(A, B, C, G, H, I)$
  - Dependencies:
    $A \rightarrow B$, $A \rightarrow C$, $CG \rightarrow H$, $CG \rightarrow I$, $B \rightarrow H$
  - Logically implies: $A \rightarrow H$, $CG \rightarrow HI$, $AG \rightarrow I$

Rules of Inference (2)

- Axioms are rules of inference for dependencies
- This group is called Armstrong's axioms
- Greek letters α, β, γ, … represent attribute sets
- Reflexivity rule:

  If α is a set of attributes and $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$ holds.
- Augmentation rule:

  If $\alpha \rightarrow \beta$ holds, and γ is a set of attributes, then $\gamma\alpha \rightarrow \gamma\beta$ holds.
- Transitivity rule:

  If $\alpha \rightarrow \beta$ holds, and $\beta \rightarrow \gamma$ holds, then $\alpha \rightarrow \gamma$ holds.

Computing Closure of F

Can use Armstrong's axioms to compute F^+ from F

- F is a set of functional dependencies

$F^+ = F$
repeat
  for each functional dependency f in F^+
    apply reflexivity and augmentation rules to f
    add resulting functional dependencies to F^+
  for each pair of functional dependencies f_1 , f_2 in F^+
    if f_1 and f_2 can be combined using transitivity
      add resulting functional dependency to F^+
until F^+ stops changing

Armstrong's Axioms

- Axioms are <u>sound</u>
  - They don't generate any incorrect functional dependencies
- Axioms are <u>complete</u>
  - Given a set of functional dependencies F, repeated application generates all F^+
- F^+ could be <u>very</u> large
  - LHS and RHS of a dependency are subsets of R
  - A set of size n has 2^n subsets
  - $2^n \times 2^n = 2^{2n}$ possible functional dependencies in R !

More Rules of Inference

- Additional rules can be proven from Armstrong's axioms
  - These make it easier to generate F^+
- Union rule:

  If $\alpha \rightarrow \beta$ holds, and $\alpha \rightarrow \gamma$ holds, then $\alpha \rightarrow \beta\gamma$ holds.
- Decomposition rule:

  If $\alpha \rightarrow \beta\gamma$ holds, then $\alpha \rightarrow \beta$ holds and $\alpha \rightarrow \gamma$ holds.
- Pseudotransitivity rule:

  If $\alpha \rightarrow \beta$ holds, and $\gamma\beta \rightarrow \delta$ holds, then $\alpha\gamma \rightarrow \delta$ holds.

Attribute-Set Closure

- How to tell if an attribute-set α is a superkey?
  - If $\alpha \rightarrow R$ then α is a superkey.
  - What attributes are functionally determined by an attribute-set α ?
- Given:
  - Attribute-set α
  - Set of functional dependencies F
  - The set of all attributes functionally determined by α under F is called the closure of α under F
  - Written as α^+

Attribute-Set Closure (2)

- It's easy to compute the closure of attribute-set α !
  - Algorithm is very simple
- Inputs:
  - attribute-set α
  - set of functional dependencies *F*

$\alpha^+ = \alpha$
repeat
    for each functional dependency $\beta \rightarrow \gamma$ in *F*
        if $\beta \subseteq \alpha^+$ **then** $\alpha^+ = \alpha^+ \cup \gamma$
until α^+ stops changing

Attribute-Set Closure (3)

- Can easily test if α is a superkey
  - Compute α^+
  - If $R \subseteq \alpha^+$ then α is a superkey of R
- Can also use to identify functional dependencies
  - $\alpha \rightarrow \beta$ holds if $\beta \subseteq \alpha^+$
    - Find closure of α under F; if it contains β then $\alpha \rightarrow \beta$ holds!
  - Can compute F^+ with attribute-set closure too:
    - For each $\gamma \subseteq R$, find closure γ^+ under F
      - We know that $\gamma \rightarrow \gamma^+$
    - For each subset $S \subseteq \gamma^+$, add functional dependency $\gamma \rightarrow S$

Attribute-Set Closure Example

- Relation schema $R(A, B, C, G, H, I)$
  - Dependencies:
    $A \rightarrow B$, $A \rightarrow C$, $CG \rightarrow H$, $CG \rightarrow I$, $B \rightarrow H$
- Is AG a superkey of R ?
- Compute $(AG)^+$
  - Start with $\alpha^+ = AG$
  - $A \rightarrow B$, $A \rightarrow C$ cause $\alpha^+ = ABCG$
  - $CG \rightarrow H$, $CG \rightarrow I$ cause $\alpha^+ = ABCGHI$
- AG is a superkey of R !

Attribute-Set Closure Example (2)

- Relation schema $R(A, B, C, G, H, I)$
  - Dependencies:
    $A \rightarrow B$, $A \rightarrow C$, $CG \rightarrow H$, $CG \rightarrow I$, $B \rightarrow H$
- Is AG a candidate key of R ?
  - A candidate key is a minimal superkey
  - Compute attribute-set closure of all proper subsets of superkey; if we get R then it's not a candidate key
- Compute the attribute-set closures under F
  - $A^+ = ABCH$
  - $G^+ = G$
- AG is indeed a candidate key!

BCNF Revisited

- BCNF algorithm states, if R_i is a schema not in BCNF:
  - There is at least one nontrivial functional dependency $\alpha \rightarrow \beta$ such that α is not a superkey for R_i
- Two points:
  - $\alpha \rightarrow \beta \in F^+$, not just in F
  - For R_i, only care about func. deps. where $\alpha \cup \beta \in R_i$
- How do we tell if R_i is not in BCNF?
  - Can use attribute-set closure under F to find if there is a dependency in F^+ that affects R_i
  - For each proper subset $\alpha \subset R_i$, compute α^+ under F
  - If α^+ doesn't contain R_i , but α^+ <u>does</u> contain any attributes in $R_i - \alpha$, then R_i is <u>not</u> in BCNF

BCNF Revisited (2)

- If α^+ doesn't contain R_i, but α^+ does contain any attributes in $R_i - \alpha$, then R_i is not in BCNF
- If α^+ doesn't contain R_i, what do we know about α with respect to R_i ?
  - α is not a superkey of R_i
- If α^+ contains attributes in $R_i - \alpha$:
  - Let $\beta = R_i \cap (\alpha^+ - \alpha)$
  - We know there is some non-trivial functional dependency $\alpha \rightarrow \beta$ that holds on R_i
- Since $\alpha \rightarrow \beta$ holds on R_i, but α is not a candidate key of R_i, we know that R_i cannot be in BCNF.

BCNF Example

- Start with schema $R(A, B, C, D, E)$, and $F = \{ A \rightarrow B, BC \rightarrow D \}$
- Is R in BCNF?
  - Obviously not.
  - Using $A \rightarrow B$, decompose into $R_1(\underline{A}, B)$ and $R_2(A, C, D, E)$
- Are we done?
  - Pseudotransitivity rule says that if $\alpha \rightarrow \beta$ and $\gamma\beta \rightarrow \delta$, then $\alpha\gamma \rightarrow \delta$
  - $AC \rightarrow D$ also holds on R_2, so R_2 is not in BCNF!
  - Or, compute $\{AC\}^+ = ABCD$. Again, R_2 is not in BCNF.

Database Constraints

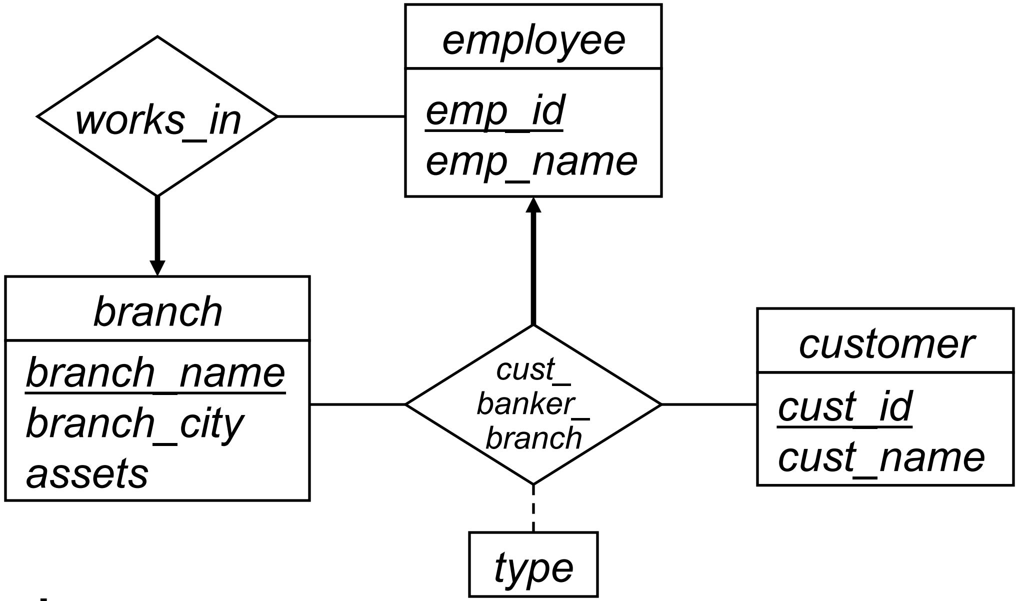
- Enforcing database constraints can easily become very expensive
  - Especially **CHECK** constraints!
- Best to define database schema such that constraint enforcement is <u>efficient</u>
- Ideally, enforcing a functional dependency involves only one relation
  - Then, can specify a key constraint instead of a multi-table **CHECK** constraint!

Example: Personal Bankers

- Bank sets a requirement on employees:
  - Each employee can work at only one branch
  - *emp_id* $\rightarrow$ *branch_name*
- Bank wants to give customers a personal banker at each branch
  - At each branch, a customer has only one personal banker
  - (A customer could have personal bankers at multiple branches.)
  - *cust_id*, *branch_name* $\rightarrow$ *emp_id*

Personal Bankers

- E-R diagram:



- Relationship-set schemas:

  works_in(*emp_id*, *branch_name*)

  cust_banker_branch(*cust_id*, *branch_name*, *emp_id*, *type*)

Personal Bankers (2)

- Schemas:

  works_in(<u>*emp_id*</u>, *branch_name*)

  cust_banker_branch(<u>*cust_id*</u>, <u>*branch_name*</u>, *emp_id*, *type*)
- Is this schema in BCNF?
  - *emp_id* → *branch_name*
  - *cust_banker_branch* isn't in BCNF
    - *emp_id* isn't a candidate key on *cust_banker_branch*
  - *cust_banker_branch* repeats *branch_name* unnecessarily, since *emp_id* → *branch_name*
- Decompose into two BCNF schemas:
  - *works_in* already has (<u>*emp_id*</u>, *branch_name*) $(\alpha \cup \beta)$
  - Create *cust_banker*(*cust_id*, *emp_id*, *type*) $(R - (\beta - \alpha))$

Personal Bankers (3)

- New BCNF schemas:

  works_in(<u>*emp_id*</u>, *branch_name*)

  cust_banker(<u>*cust_id*</u>, <u>*emp_id*</u>, *type*)

  - A customer can have one personal banker at each branch, so both *cust_id* and *emp_id* must be in the primary key
- Any problems with this new BCNF version?
  - Now we can't <u>easily</u> constrain that each customer has only one personal banker at each branch!
  - Could still create a complicated **`CHECK`** constraint involving multiple tables…

Preserving Dependencies

- The BCNF decomposition doesn't preserve this dependency:
  - *cust_id, branch_name* → *emp_id*
  - Can't enforce this dependency within a single table
- In general, BCNF decompositions are not <u>dependency-preserving</u>
  - Some functional dependencies are not enforceable within a single table
  - Can't enforce them with a simple key constraint, so they are more expensive
- Solution: Third Normal Form

Third Normal Form

- Slightly weaker than Boyce-Codd normal form
  - Preserves more functional dependencies
  - Also allows more repeated information!
- Given:
  - Relation schema *R*
  - Set of functional dependencies *F*
- *R* is in 3NF with respect to *F* if:
  - For all functional dependencies $\alpha \rightarrow \beta$ in F^+, where $\alpha \subseteq R$ and $\beta \subseteq R$, at least one of the following holds:
    - $\alpha \rightarrow \beta$ is a trivial dependency
    - α is a superkey for *R*
    - Each attribute *A* in $\beta - \alpha$ is contained in a candidate key for *R*

Third Normal Form (2)

- New condition:
  - Each attribute *A* in $\beta - \alpha$ is contained in a candidate key for *R*
- A general constraint:
  - Doesn't require a single candidate key to contain all attributes in $\beta - \alpha$
  - Just requires that each attribute in $\beta - \alpha$ appears in *some* candidate key in *R*
  - …possibly even different candidate keys!

Personal Banker Example

- Our non-BCNF personal banker schemas again:
  - *works_in*(*<u>emp_id</u>*, *branch_name*)
  - *cust_banker_branch*(*<u>cust_id</u>*, *<u>branch_name</u>*, *emp_id*, *type*)
- Is this schema in 3NF?
  - *emp_id* → *branch_name*
  - *cust_id*, *branch_name* → *emp_id*
- *works_in* is in 3NF (*emp_id* is the primary key)
- What about *cust_banker_branch* ?
  - Both dependencies hold on *cust_banker_branch*
    - *emp_id* → *branch_name*, but *emp_id* isn't the primary key
    - *cust_id*, *branch_name* → *emp_id* ; is *emp_id* part of any candidate key on *cust_banker_branch* ?

Personal Banker Example (2)

- Look carefully at the functional dependencies:
  - Primary key of *cust_banker_branch* is (*cust_id*, *branch_name*)
    - { *cust_id*, *branch_name* } $\rightarrow$ *cust_banker_branch* (all attributes) (constraint arises from the E-R diagram & schema translation)
    - (Also specified this constraint: *cust_id*, *branch_name* $\rightarrow$ *emp_id*)
  - We also know that *emp_id* $\rightarrow$ *branch_name*
  - Pseudotransitivity rule: if $\alpha \rightarrow \beta$ and $\gamma\beta \rightarrow \delta$, then $\alpha\gamma \rightarrow \delta$
    - { *emp_id* } $\rightarrow$ { *branch_name* }
    - { *cust_id*, *branch_name* } $\rightarrow$ *cust_banker_branch*
    - Therefore, { *emp_id*, *cust_id* } $\rightarrow$ *cust_banker_branch* also holds!
  - (*cust_id*, *emp_id*) is a candidate key of *cust_banker_branch*
- So *cust_banker_branch* is in fact in 3NF
  - *(And we need to enforce this second candidate key too…)*

Canonical Cover

- Given a relation schema, and a set of functional dependencies *F*
- Database needs to enforce *F* on all relations
  - Invalid changes should be rolled back
- *F* could contain a lot of functional dependencies
  - Dependencies might even logically imply each other
- Want a minimal version of *F*, that still represents all constraints imposed by *F*
  - Should be more efficient to enforce minimal version

Canonical Cover (2)

- A canonical cover F_c for F is a set of functional dependencies such that:
  - F logically implies all dependencies in F_c
  - F_c logically implies all dependencies in F
  - Can't infer any functional dependency in F_c from other dependencies in F_c
  - No functional dependency in F_c contains an extraneous attribute
  - Left side of all functional dependencies in F_c are unique
    - There are no two dependencies $\alpha_1 \rightarrow \beta_1$ and $\alpha_2 \rightarrow \beta_2$ in F_c such that $\alpha_1 = \alpha_2$

Extraneous Attributes

- Given a set of functional dependencies F
  - An attribute in a functional dependency is <u>extraneous</u> if it can be removed from F without affecting closure of F
- Formally: given F, and $\alpha \to \beta$
  - If $A \in \alpha$, and F logically implies $(F - \{\alpha \to \beta\}) \cup \{(\alpha - A) \to \beta\}$, then A is extraneous
  - If $A \in \beta$, and $(F - \{\alpha \to \beta\}) \cup \{\alpha \to (\beta - A)\}$ logically implies F, then A is extraneous
    - i.e. generate a new set of functional dependencies F' by replacing $\alpha \to \beta$ with $\alpha \to (\beta - A)$
    - See if F' logically implies F

Testing Extraneous Attributes

- Given relation schema *R*, and a set *F* of functional dependencies that hold on *R*
- Attribute *A* in $\alpha \rightarrow \beta$
- If $A \in \alpha$ (i.e. *A* is on left side of the dependency), then let $\gamma = \alpha - \{A\}$
  - See if $\gamma \rightarrow \beta$ can be inferred from *F*
  - Compute γ^+ under *F*
  - If $\beta \subseteq \gamma^+$, then *A* is extraneous in α

Testing Extraneous Attributes (2)

- Given relation schema *R*, and a set *F* of functional dependencies that hold on *R*
- Attribute *A* in $\alpha \rightarrow \beta$
- If $A \in \beta$ (on right side of the dependency), then try the <u>altered</u> set F'
  - $F' = (F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$
  - See if $\alpha \rightarrow A$ can be inferred from F'
  - Compute α^+ under F'
  - If α^+ includes *A*, then *A* is extraneous in β

Computing Canonical Cover

- A simple way to compute the canonical cover of F

  $F_c = F$
  repeat
  apply union rule to replace dependencies in F_c of form
  $\alpha_1 \rightarrow \beta_1$ and $\alpha_1 \rightarrow \beta_2$ with $\alpha_1 \rightarrow \beta_1\beta_2$
  find a functional dependency $\alpha \rightarrow \beta$ in F_c with an
  extraneous attribute
  /* Use F_c for the extraneous attribute test, not F !!! */
  if an extraneous attribute is found, delete it from $\alpha \rightarrow \beta$
  until F_c stops changing

Canonical Cover Example

- Functional dependencies F on schema (A, B, C)
  - $F = \{ A \rightarrow BC, B \rightarrow C, A \rightarrow B, AB \rightarrow C \}$
  - Find F_c
- Apply union rule to $A \rightarrow BC$ and $A \rightarrow B$
  - Left with: $\{ A \rightarrow BC, B \rightarrow C, AB \rightarrow C \}$
- A is extraneous in $AB \rightarrow C$
  - $B \rightarrow C$ is logically implied by F (obvious)
  - Left with: $\{ A \rightarrow BC, B \rightarrow C \}$
- C is extraneous in $A \rightarrow BC$
  - Logically implied by $A \rightarrow B, B \rightarrow C$
- $F_c = \{ A \rightarrow B, B \rightarrow C \}$

Another Example

- Functional dependencies F on schema (A, B, C, D)
  - $F = \{ A \rightarrow B, BC \rightarrow D, AC \rightarrow D \}$
  - Find F_c
- In this case, it may look like $F_c = F$…
- However, can infer $AC \rightarrow D$ from $A \rightarrow B$, $BC \rightarrow D$ (pseudotransitivity), so $AC \rightarrow D$ is extraneous in F
  - Therefore, $F_c = \{ A \rightarrow B, BC \rightarrow D \}$
- Alternately, can argue that D is extraneous in $AC \rightarrow D$
  - With $F' = \{ A \rightarrow B, BC \rightarrow D \}$, we see that $\{AC\}^+ = ACD$, so D is extraneous in $AC \rightarrow D$
  - (If you eliminate the entire RHS of a functional dependency, it goes away)

Canonical Covers

- A set of functional dependencies can have multiple canonical covers!
- Example:
  - $F = \{ A \rightarrow BC, B \rightarrow AC, C \rightarrow AB \}$
  - Has several canonical covers:
    - $F_c = \{ A \rightarrow B, B \rightarrow C, C \rightarrow A \}$
    - $F_c = \{ A \rightarrow B, B \rightarrow AC, C \rightarrow B \}$
    - $F_c = \{ A \rightarrow C, C \rightarrow B, B \rightarrow A \}$
    - $F_c = \{ A \rightarrow C, B \rightarrow C, C \rightarrow AB \}$
    - $F_c = \{ A \rightarrow BC, B \rightarrow A, C \rightarrow A \}$