## NORMAL FORMS

## CS121: Relational Databases

## Equivalent Schemas

$\square$ Many different schemas can represent a set of data
$\square$ Which one is best?
$\square$ What does "best" even mean?
$\square$ Main goals:
$\square$ Representation must be complete
$\square$ Data should not be unnecessarily redundant
$\square$ Should be easy to manipulate the information
$\square$ Should be easy to enforce [most] constraints

## Normal Forms

$\square$ A "good" pattern for database schemas to follow is called a normal form
$\square$ Several different normal forms, with different constraints
$\square$ Normal forms can be formally specified
$\square$ Can test a schema against a normal form

- Can transform a schema into a normal form
$\square$ Goal:
$\square$ Design schemas that satisfy a particular normal form
$\square$ If a schema isn't "good," transform it into an appropriate normal form


## Example Schema Design

$\square$ Schema for representing loans and borrowers:
$\square$ customer relation stores customer details, including a cust_id primary-key attribute

- loan(loan id, amount)
- borrower(cust_id, loan_id)
$\square$ Many-to-many mapping

| loan_id | amount |
| :--- | :--- |
| $\ldots$ | $\ldots$ |
| L-100 | 10000 |
| $\ldots$ | $\ldots$ |
|  | Ioan |

$\square$ A customer can have multiple loans
$\square$ A loan can be owned by multiple customers

| cust_id | loan_id |
| :--- | :--- |
| $\ldots$ | $\ldots$ |
| $23-652$ | L-100 |
| $15-202$ | L-100 |
| $23-521$ | L-100 |
| $\ldots$ | $\ldots$ |
| borrower |  |

## Larger Schema?

$\square$ Could replace loan and borrower relations with a larger, combined relation bor_loan(cust id, loan id, amount)
$\square$ Rationale:

| cust_id | loan_id | amount |
| :--- | :--- | :--- |
| $\ldots$ | $\ldots$ | $\ldots$ |
| $23-652$ | L-100 | 10000 |
| $15-202$ | L-100 | 10000 |
| $23-521$ | L-100 | 10000 |
| $\ldots$ | $\ldots$ | $\ldots$ |
| bor_loan |  |  |

$\square$ Eliminates a join when retrieving loan amounts
$\square$ Problem: mapping between customers and loans is many-to-many
$\square$ Multiple redundant copies of amount to keep in sync!

## Repeated Values

$\square$ How do we know that this is a problem?

- "Because we see values that appear multiple times"
$\square$ This isn't a good enough reason!!!
$\square$ Could easily have different loans with the same amount
$\square$ A repeated value doesn't automatically indicate a problem...

| cust_id | loan_id | amount |
| :--- | :--- | :--- |
| $\ldots$ | $\ldots$ | $\ldots$ |
| $23-652$ | L-100 | 10000 |
| $19-065$ | L-205 | 10000 |
| $15-202$ | L-100 | 10000 |
| $23-521$ | L-100 | 10000 |
| $20-419$ | L-205 | 10000 |
| $\ldots$ | $\ldots$ | $\ldots$ |
| bor_loan |  |  |

## Back to the Enterprise

$\square$ What are the rules of the enterprise that we are modeling?

- "Every loan must have only one amount."
$\square$ In other words:
$\square$ Every loan ID corresponds to exactly one amount.
- If there were a schema (loan_id, amount) then loan_id can be a primary key.
$\square$ Specified as a functional dependency
- loan_id $\rightarrow$ amount
- loan_id functionally determines amount


## Repeated Values v2.0

$\square$ bor_loan relation has both loan_id and amount attributes bor_loan(cust id, loan id, amount)
$\square$ But, loan_id $\rightarrow$ amount, and loan_id by itself can't be a primary key in bor_loan
$\square$ Need to support many-to-many mappings between customers and loans

- Combination of cust_id and loan_id must be a primary key, so a particular loan_id value can appear multiple times
$\square$ In rows with the same loan_id value, amount will have to be repeated.


## Functional Dependencies

$\square$ Functional dependencies are very important in schema analysis
$\square$ Have a lot to do with keys!

- "Good" schema designs are guided by functional dependencies
$\square$ Frequently helpful to identify them during schema design
$\square$ Can formally define functional dependencies, and reason about them
$\square$ Can also specify constraints on schemas using functional dependencies


## Another Example Schema

$\square$ A "large" schema for employee information employee(emp id, emp_name, phone, title, salary, start_date)

| emp_id | emp_name | phone | title | salary | start_date |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |  |
| $123-45-6789$ | Jeff | $555-1234$ | CTO | 120000 | $1996-03-15$ |  |
| $314-15-9265$ | Mary | $555-3141$ | CFO | 120000 | $1997-08-02$ |  |
| $987-65-4321$ | Helen | $555-9876$ | Developer | 90000 | $1996-05-23$ |  |
| $101-01-0101$ | Marcus | $555-1010$ | Tester | 70000 | $1995-11-04$ |  |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |  |  |
| employee |  |  |  |  |  |  |

$\square$ Employee ID is unique, but other attributes could have duplicate values

## Smaller Schemas?

$\square$ Could represent this with two smaller schemas:
emp_ids(emp id, emp_name)
emp_details(emp_name, phone, title, salary, start_date)

| emp_id | emp_name |
| :--- | :--- |
| $\ldots$ | $\ldots$ |
| $123-45-6789$ | Jeff |
| $314-15-9265$ | Mary |
| $987-65-4321$ | Helen |
| $101-01-0101$ | Marcus |
| $\ldots$ | $\ldots$ |
| emp ids |  |


| emp_name | phone | title | salary | start_date |
| :--- | :--- | :--- | :--- | :--- |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| Jeff | $555-1234$ | CTO | 120000 | $1996-03-15$ |
| Mary | $555-3141$ | CFO | 120000 | $1997-08-02$ |
| Helen | $555-9876$ | Developer | 90000 | $1996-05-23$ |
| Marcus | $555-1010$ | Tester | 70000 | $1995-11-04$ |
| $\ldots$ | $\ldots$ | $\ldots$ |  | $\ldots$ |
| emp_details |  |  |  |  |

$\square$ Generate original employee data with a join: emp_ids $\bowtie$ emp_details
$\square$ Any problems with this?

## emp_name is not unique!

$\square$ Joins using emp_name can generate invalid tuples!

| emp_id | emp_name |
| :--- | :--- |
| $\ldots$ | $\ldots$ |
| $314-15-9265$ | Mary |
| $161-80-3398$ | Mary |
| $\ldots$ | $\ldots$ |
| emp_ids |  |


| emp_name | phone | title | salary | start_date |
| :--- | :--- | :--- | :--- | :--- |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| Mary | $555-3141$ | CFO | 120000 | $1997-08-02$ |
| Mary | $555-1618$ | Gofer | 25000 | $1998-01-07$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |  |


| emp_id | emp_name | phone | title | salary | start_date |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $314-15-9265$ | Mary | $555-1618$ | Gofer | 25000 | $1998-01-07$ |
| $314-15-9265$ | Mary | $555-3141$ | CFO | 120000 | $1997-08-02$ |
| $161-80-3398$ | Mary | $555-3141$ | CFO | 120000 | $1997-08-02$ |
| $161-80-3398$ | Mary | $555-1618$ | Gofer | 25000 | $1998-01-07$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |  | $\ldots$ |

## Bad Decompositions

$\square$ This decomposition is clearly broken

- It can't represent the information correctly!
$\square$ Problem: enterprise needs to support different employees with the same name
$\square$ Lossy decompositions cannot accurately represent all facts about an enterprise
$\square$ Lossless decompositions can accurately represent all facts
$\square$ "Good" schema designs avoid lossy decompositions


## First Normal Form

$\square$ A schema is in first normal form (1NF) if all attribute domains are atomic
$\square$ An atomic domain has values that are indivisible units
$\square$ E-R model supports non-atomic attributes

- Multivalued attributes
$\square$ Composite attributes
$\square$ Relational model specifies atomic domains for attributes
$\square$ Schemas are automatically in 1NF
$\square$ Mapping from E-R model to relational model changes composite/multivalued attributes into an atomic form


## 1 NF Example

$\square$ E-R diagram for magazine subscribers
$\square$ address is composite
$\square$ email_addr is multivalued

| subscriber |
| :---: |
| sub id |
| \{ email_addr $\}$ |
| address |
| street |
| city |
| state |
| zip_code |

$\square$ Converts to a 1NF schema:
subscriber(sub id, street, city, state, zip_code) sub_emails(sub id, email addr)
$\square$ The conversion rules we have discussed, automatically convert E-R schemas into 1 NF

## 1 NF and Non-Atomic Attributes

$\square$ Many, but not all, SQL DBs have non-atomic types
$\square$ Some offer support for composite attributes
$\square$ Some offer support for multivalued attributes

- These are SQL extensions - not portable
$\square$ As long as you steer clear of using non-atomic attributes in primary/foreign keys, can sometimes be quite useful
$\square$ Will likely encounter them very rarely in practice, though
$\square$ Biggest reason: DB support for list/vector column-types isn't terribly widespread, or always very easy to use


## 1 NF and Non-Atomic Attributes (2)

$\square$ Composite types:
$\square$ e.g. defining an "address" composite type
$\square$ Can definitely be useful for making a schema clearer, as long as they aren't used in a key!
$\square$ Multivalued types:
$\square$ e.g. arrays, lists, sets, vectors
$\square$ Can sometimes be useful for storing pre-computed values that aren't expected to change frequently

- If you are regularly issuing queries that search through or change these values, you may need to revise your schema!
- Should probably factor non-atomic data out into a separate table


## Other Normal Forms

$\square$ Other normal forms relate to functional dependencies
$\square$ Analysis of functional dependencies shows if a schema needs decomposed
$\square$ Keys are functional dependencies too!
$\square$ Formally define functional dependencies, and reason about them
$\square$ Define normal forms in terms of functional dependencies

## Schemas and Constraints

$\square$ Keys and functional dependencies are constraints that a database must satisfy
$\square$ Legal relations satisfy the required constraints
$\square$ Relation doesn't contain any tuples that violate the specified constraints
$\square$ More terminology:
$\square$ Relation schema $R$, relation $r(R)$
$\square$ A set of functional dependencies $F$
$\square$ Relation $r$ satisfies $F$ if $r$ is legal
$\square$ When we say " $F$ holds on $R$ ", specifies the set of relations with $R$ as their schema, that are legal with respect to $F$

## Functional Dependencies

$\square$ Formal definition of a functional dependency:
$\square$ Given a relation schema $R$ with attribute-sets $\alpha, \beta \subseteq R$
$\square$ The functional dependency $\alpha \rightarrow \beta$ holds on $r(R)$ if $\left\langle\forall t_{1}, t_{2} \in r: t_{1}[\alpha]=t_{2}[\alpha]: t_{1}[\beta]=t_{2}[\beta]\right\rangle$
$\square$ In other words:
$\square$ For all pairs of tuples $t_{1}$ and $t_{2}$ in $r_{\text {, }}$ if $t_{1}[\alpha]=t_{2}[\alpha]$ then $t_{1}[\beta]=t_{2}[\beta]$
$\square \alpha$ functionally determines $\beta$

## Dependencies and Superkeys

$\square$ Given relation schema $R$, a subset $K$ of $R$ can be a superkey
$\square$ In a relation $r(R)$, no two tuples can share the same values for attributes in $K$
$\square$ Can also say: $K$ is a superkey if $K \rightarrow R$
$\square$ The functional dependency $K \rightarrow R$ holds if

$$
\left\langle\forall t_{1}, t_{2} \in r(R): t_{1}[K]=t_{2}[K]: t_{1}[R]=t_{2}[R]\right\rangle
$$

$\square t_{1}[R]=t_{2}[R]$ (or $t_{1}=t_{2}$ ) means $t_{1}$ and $t_{2}$ are the same tuple
$\square$ The superkey $K$ functionally determines the whole relation $R$
$\square$ Functional dependencies are a more general form of constraint than superkeys are.

## The bor_loan Relation

$\square$ bor_loan(cust_id, loan_id, amount)
$\square$ Functional dependency: loan_id $\rightarrow$ amount

- "Every loan has exactly one amount."
$\square$ Every tuple in bor_loan with a given loan_id value must have the same amount value
$\square$ bor_loan also has a primary key
$\square$ Specifies another functional dependency
$\square$ cust_id, loan_id $\rightarrow$ cust_id, loan_id, amount
$\square$ This is not a functional dependency specifically required by what the enterprise needs to model
- Can be inferred from other functional dependencies in the schema


## Trivial Dependencies

$\square$ A trivial functional dependency is satisfied by all relation values!
$\square$ For a relation $R$ containing attributes $A$ and $B$, $A \rightarrow A$ is a trivial dependency
$\left\langle\forall t_{1}, t_{2} \in r: t_{1}[A]=t_{2}[A]: t_{1}[A]=t_{2}[A]\right\rangle$

- Well, duh!
$\square A B \rightarrow A$ is also a trivial dependency
- If $t_{1}[A B]=t_{2}[A B]$, then of course $t_{1}[A]=t_{2}[A]$ too!
$\square$ In general: $\alpha \rightarrow \beta$ is trivial if $\beta \subseteq \alpha$


## Closure

$\square$ Given a set of functional dependencies, we can infer other dependencies
$\square$ Given relation schema $R(A, B, C)$
$\square$ If $A \rightarrow B$ and $B \rightarrow C$, holds on $R$, then $A \rightarrow C$ also holds on $R$
$\square$ Given a set of functional dependencies $F$
$\square F^{+}$denotes the closure of $F$
$\square F^{+}$includes $F$, and all dependencies that can be inferred from $F$. $\left(F \subseteq F^{+}\right)$

## Boyce-Codd Normal Form

$\square$ Eliminates all redundancy that can be discovered using functional dependencies
$\square$ Given:
$\square$ Relation schema $R$
$\square$ Set of functional dependencies $F$
$\square R$ is in BCNF with respect to $F$ if:
$\square$ For all functional dependencies $\alpha \rightarrow \beta$ in $F^{+}$, where $\alpha \subseteq R$ and $\beta \subseteq R$, at least one of the following holds:
$\square \alpha \rightarrow \beta$ is a trivial dependency
$\square \alpha$ is a superkey for $R$
$\square$ A database design is in BCNF if all schemas in the design are in BCNF

## BCNF Examples

$\square$ The bor_loan schema isn't in BCNF bor_loan(cust_id, loan_id, amount)
$\square$ loan_id $\rightarrow$ amount holds on bor_loan
$\square$ This is not a trivial dependency, and loan_id isn't a superkey for bor_loan
$\square$ The borrower and loan schemas are in BCNF borrower(cust_id, loan_id)
$\square$ No nontrivial dependencies hold loan(loan_id, amount)
$\square$ loan_id $\rightarrow$ amount holds on loan

- loan_id is the primary key of loan


## BCNF Decomposition

$\square$ If $R$ is a schema not in BCNF:
$\square$ There is at least one nontrivial functional dependency $\alpha \rightarrow \beta$ such that $\alpha$ is not a superkey for $R$
$\square$ Replace $R$ with two schemas:
$(\alpha \cup \beta)$
$(R-(\beta-\alpha))$

- (stated this way in case $\alpha$ and $\beta$ overlap; usually they don't)
$\square$ The new schemas might also not be in BCNF!
$\square$ Repeat this decomposition process until all schemas are in BCNF


## Undoing the Damage

$\square$ For bor_loan, $\alpha=$ loan_id, $\beta=$ amount

$$
\begin{aligned}
& R=\text { (cust_id, loan_id, amount) } \\
& (\alpha \cup \beta)=(\text { loan_id, amount }) \\
& (R-(\beta-\alpha))=(\text { cust_id, loan_id })
\end{aligned}
$$

$\square$ Rules successfully decompose bor_loan back into loan and borrower schemas

## Review

$\square$ Normal forms are guidelines for what makes a database design "good"
$\square$ Can formally specify them
$\square$ Can transform schemas into normal forms
$\square$ Functional dependencies specify constraints between attributes in a schema
$\square$ A more general kind of constraint than key constraints
$\square$ Covered 1 NF and BCNF

- 1NF requires all attributes to be atomic
$\square$ BCNF uses functional dependencies to eliminate redundant data


## Next Time!

$\square$ A big question to explore:
$\square$ Given a set of functional dependencies $F$, we need to know what dependencies can be inferred from it!

- i.e. given $F$, how to compute $F^{+}$
$\square$ BCNF needs this information, as do other normal forms
$\square$ Does Boyce-Codd Normal Form have drawbacks?
$\square$ (yes.)
$\square$ Motivates the development of $3^{\text {rd }}$ Normal Form

